

## REGIONAL FREQUENCY ANALYSIS OF EXTREME RAINFALL IN THE UMBRIA REGION (CENTRAL ITALY)

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Regional Frequency Analysis (RFA) is often used in hydrology to obtain rainfall estimates in places short data series. In this work RFA has been applied to extreme rainfall data series (with durations from 1 to 24 hours), with 23 rain gauges at the Umbria Region (Central Italy). The most important step in RFA is the creation of homogeneous groups of stations. The multifractal behaviour of hourly rainfall has resulted to be a very useful characteristic when joining stations into regions. Thus, a multifractal analysis of the hourly rainfall data series available for the 23 stations (from 1992 to 2013) has been carried out and the empirical moments scaling exponent functions  $K(q)$  have been obtained. Two parameters from the  $K(q)$  functions have been the bases for grouping the stations into regions when necessary. From the heterogeneity test of RFA, it has been checked that all the 23 stations form a homogeneous region for all the durations except than for 6 hours. A cluster analysis by means of multifractal parameters has then been performed for this duration. Two homogeneous subregions have been obtained with only one site excluded, being the local frequency analysis the one to be used for this place.

**Keywords:** extreme rainfall; regional frequency analysis; multifractality

## ANÁLISIS REGIONAL DE FRECUENCIAS DE LLUVIAS EXTREMAS EN LA REGIÓN DE UMBRÍA (ITALIA CENTRAL)

La metodología del Análisis Regional de Frecuencias (ARF) se utiliza con frecuencia en hidrología para obtener cuantiles de precipitación en lugares donde las series de datos disponibles son de corta duración. En este trabajo se ha realizado un ARF de precipitaciones extremas (para duraciones desde 1 a 24 horas) con datos de 23 estaciones de la región de Umbría (Italia central). El paso más importante del ARF es la formación de regiones homogéneas, siendo la caracterización multifractal de la lluvia muy útil para dicha finalidad. Por lo tanto, se ha llevado a cabo el análisis multifractal de las series horarias de lluvia disponibles, obteniéndose la función exponente empírica escaladora de momentos,  $K(q)$ , y dos de sus parámetros más relevantes que serán la base a la hora de agrupar las estaciones en regiones. Tras aplicar el test de heterogeneidad para las distintas duraciones, se ha comprobado que, excepto para 6 horas, todas las estaciones forman una región homogénea. Para 6 horas y usando los parámetros multifractales previamente obtenidos, se han formado dos regiones homogéneas en las que solo una de las 23 estaciones ha quedado excluida del análisis, siendo el análisis local de frecuencias el más adecuado para esta localidad.

**Palabras clave:** Lluvia extrema; análisis regional de frecuencias; multifractalidad

## 1. INTRODUCTION

Rainfall is one of the most complex atmospheric processes that directly affects human activities. The knowledge of extreme rainfall intensities for a certain return period is a crucial problem for hydraulic work designers. The classical approach to facing this issue is based on an estimation of the intensity-duration-frequency (IDF) curves, which let one compute the expected rainfall for a given occurrence probability and duration (Di Baldassarre et al., 2006). These curves are calculated starting from the annual maximum rainfall data series of different durations for a fixed weather station. The application of probabilistic models, based on the extreme value theory to these series allows one to compute the quantile estimations for different return periods used to obtain the parameters of the equation of the IDF curve chosen among the different formulations provided by the literature (Chow, 1964; Bell, 1969; Chen, 1983; T mez, 1987).

The case of the presence of a data series with too limited a number of records to permit a reliable statistical analysis is very common and the estimation of the expected rainfall for a fixed probability becomes very complicated. In these cases, a Regional Frequency Analysis (RFA) could help to overcome the problem. This methodology is very useful for obtaining more accurate quantile estimations than the Local Frequency Analysis (Hosking et al., 1985; Lettenmaier and Potter, 1985; Wallis and Wood, 1985) and it is applicable only if the study area can be divided into homogeneous regions. The determination of homogeneous regions could be very complex and different methodologies can be adopted like: the cluster analysis (Periago et al., 1991; Bonell and Sumner, 1992), the principal component analysis (Garc a-Mar n et al., 2011); Fuzzy C-means clustering combined with artificial neural networks (Srinivas et al., 2008; Satyanarayana and Srinivas, 2011); and the visual inspection of L-Moments diagrams (Y rekli and Modarres, 2007). All these techniques group together different rainfall stations using statistical values like extreme daily annual rainfall data (Garc a-Mar n et al., 2011), parameters from the probability distribution functions of the data (Easterling, 1989), multifractal analysis parameters (Garc a-Mar n et al., 2015) or other variables like latitude, longitude and altitude (Guttman, 1993), distance to the sea (Moreno and Rold n, 1999), atmospheric characteristics (Satyanarayana and Srinivas, 2011). Linear moments can be used when RFA is applied (Hosking and Wallis, 1993, 1995, 1997; Rao and Hamed, 2000).

In the last decades rainfall analysis using multifractal processes has become very diffused, in fact many works confirm that rainfall has a typical scale invariance of between 1 hour and several days in time, and between 1 km and 100 km in space (Schertzer and Lovejoy, 1987; Lovejoy and Schertzer, 1995; de Lima and Grasman, 1999; Olsson and Burlando, 2002; Veneziano and Furcolo, 2002; Garc a-Mar n et al., 2008). The main advantage of this analysis is that the multifractal parameters are not dependent on the available data and a probability distribution to describe the data set is not necessary. Turbulence formalism is one of the multifractal methodologies most applied in hydrology (Schertzer and Lovejoy, 1987). In the latter, the empirical moments scaling exponent function  $K(q)$  is obtained and its parameters can be related to the properties of the rainfall data set (de Lima and de Lima, 2009). Furthermore, multifractal rainfall models have been used to analyse the validity of the IDF curves because they describe the intensity variation with the averaging duration of rainfall (Veneziano et al., 2006; Langousis and Veneziano, 2007; Veneziano et al., 2007; Garc a-Mar n et al., 2012).

The aim of this work was to obtain homogeneous regions over the study area by using two parameters obtained from the exponent  $K(q)$  function ( $K(0)$  and  $\gamma_{max}$ ) and then to determine the RFA curves for each station.

## 2. METHODOLOGY

### 2.1 Data quality control

Continuous hourly rainfall data series from 23 rainfall stations of the Umbria region rain gauge network have been used in the present work. A preliminary data quality control has been necessary to avoid erroneous estimations during the work's development. As a first stage a manual data validation was made: in fact, the number of missing data has been calculated for each year and the rainfall presence/absence has been checked, by examining the records from the rain gauge nearest to the investigated one. If a missing data period during a rainfall event occurred, the station was flagged as "suspect" for the studied year and "rejected" in case the event was particularly strong. A range test and a persistence test have been chosen among many meteorological data validation methodologies (Zahumensky, 2004). The range test consists of considering data valid if they are included between a lower and an upper threshold (Estévez et al., 2011b). This procedure is applicable to all the weather variables. In the case of rainfall depth, the lower limit is due to the fact that no negative measurement can exist and the upper limit is a fixed value suggested in the scientific literature (Shafer et al., 2000) and it is equal to 240 mm in 1 hour. If a value was outside this range, it was flagged as "erroneous" and removed from the hourly data series.

The persistence test focuses on the attention to the data variability. This method is based on the consideration that data could be affected by mistakes if they are simultaneously not equal to zero and their persistence is greater than three (Hubbard et al., 2005). So the following relation has to be verified:

$$P(d) \neq P(d+1) \neq P(d+2) \quad (1)$$

Where  $P$  is the rainfall record and  $d$  is the time step. When this is not satisfied the values are flagged as "suspect" and further detection on the record is needed.

### 2.2 Frequency Analysis and Multifractality

The most common approach in rainfall study is the frequency analysis, which is the estimation of how often a determined event will happen and leads to the Depth Duration Frequency (DDF) Curves and the Intensity Duration Frequency (IDF) Curves development (Koutsoyiannis et al., 1998). These curves are charts by which the relationship between rainfall Depth/Intensity, probability and durations is expressed. Following the Frequency Analysis procedure the annual extreme data series for different durations (usually 1, 3, 6, 12, 24 hours) are fitted by a theoretical probability distribution function using method of moments, maximum likelihood estimation or L-Moments (Haan 1977; Hosking et al. 1985; Martin and Stedinger, 2000) for the parameter estimation. After this stage, the fitted distribution is used to obtain the quantile estimates for different durations and for a fixed return period ( $Tr$ ), which is the most common way to express the non-exceedance probability in hydrology and is the average time between two rainfall depth/intensity values exceeding an established threshold. IDF curves permit one to find out the expected rainfall intensity for a fixed duration and fixed  $Tr$  plotting a parametric equation chosen from the ones proposed in the scientific literature (Wenzel, 1982). These curves present 2, 3 or 4 parameters which are calculated by using the quantile estimates: increasing the number of parameters [they fit in a better way which fit best?? no está claro] the rainfall quantiles in the final plot but the estimation uncertainty becomes higher (Di Baldassarre et al., 2006).

Regional Frequency Analysis (RFA) is a method applied when the sample length at a single site is not adequate for proceeding with the method described above, or when an extreme estimation is necessary in a place without rain gauges. In fact, in the case of environmental observations, the same quantities are observed in different places and the event frequencies

are often the same. So more accurate conclusions could be reached considering more data samples together, provided that the measurement sites belong to the same “region”. Following the Hosking and Wallis indications (1997), Regional Frequency Analysis is composed of 4 steps: data screening, identification of homogeneous regions, choice of a frequency distribution, and at-site quantiles estimation.

L-Moments, which were introduced by Hosking (1990, 1992) and are linear functions of the probability weighted moments (Greenwood et al., 1979), were used in the whole procedure. Data screening is an essential step to finding out whether the data are adapted to this kind of analysis and this could be done by comparing data series coming from apparently similar sites. If rainfall measurements in a fixed place are declared “discordant” from the others, then records for that site cannot be used in the RFA. In this work, the discordancy evaluation employed L-Moment Ratios (L-coefficient of variation  $L-C_V$  or  $t$ , L-skewness  $L-C_s$  or  $t_3$ , L-kurtosis  $L-C_k$  or  $t_4$ ) to obtain a singular variable  $D_i$ , which includes the presence of outliers, trends, and shifts in the mean of the sample. For each site  $i$ , one can consider the vector  $u_i = (t, t_3, t_4)$  and the group average  $\bar{u}$ , the matrix of sums of square and cross-products  $A$  and the discordancy measure  $D_i$  can be expressed as follows, with  $N$  number of samples available in the study area:

$$\bar{u} = N^{-1} \sum_{i=1}^N u_i \quad (2)$$

$$A = \sum_{i=1}^N (u_i - \bar{u})(u_i - \bar{u})^T \quad (3)$$

$$D_i = \frac{1}{3} (u_i - \bar{u})^T A^{-1} (u_i - \bar{u}) \quad (4)$$

A site is flagged as discordant if the value  $D_i$  is higher than a critical number that depends on the number of sites composing the group (Hosking and Wallis, 1997).

The second step of the RFA is the identification of homogeneous regions inside the study area, so that groups of sites satisfying homogeneity conditions are created. Cluster analysis (Periago et al., 1991; Bonell and Sumner, 1992) is one of the most practical methods and it uses data vectors to put together sites that are not necessarily geographically contiguous (Burn, 1989; Srinivas et al., 2008; Satayanarayana and Srinivas, 2011; García-Marín et al., 2015). This procedure has been used in this work and stations were put together by using site characteristics obtained from an exponent function related to the multifractal characteristics of hourly rainfall data (Guadagnini et al., 2012). The statistical moments scaling method (Shertzer and Lovejoy, 1987) is one of the most widely used methods to detect rainfall multifractality (Sivakumar, 2001). The procedure consists of analysing a record of rainfall data which is divided into different non-overlapping time intervals. The “scale ratio”  $\lambda$  is then calculated by dividing the field maximum scale for this interval, so the time is scaled so that the duration of the longest period of interest is equal to 1 (e.g. de Lima and Grasman, 1999). The scaling of the moments has to be described by using the exponent function  $K(q)$ , which satisfies (e.g. Shertzer and Lovejoy, 1987; Lovejoy and Schertzer, 1990):

$$\langle \varepsilon_\lambda^q \rangle \approx \lambda^{K(q)} \quad (5)$$

Where  $\langle \varepsilon_\lambda^q \rangle$  is the average  $q^{th}$  moment of the intensity of the process at scale  $\lambda$  and  $K(q)$  is the moments scaling exponent function. The scaling behaviour can be investigated with the log-log diagram in which it is plotted as a function of  $\lambda$ ; the resulting curve is usually linear over a certain  $\lambda$  gap and the slope is an estimate of  $K(q)$ . The complete empirical scaling moment function is obtained repeating this procedure for different values of  $q$ , and two characteristic values of it were used in this work to characterize sites and recognize homogeneous regions:  $\gamma_{max}$  and  $K(0)$ . The singularity values  $\gamma$  are upper limited by  $\gamma_{max}$ , so

the function  $K(q)$  is linear for  $q > q_{crit}$ .  $\gamma_{max}$  is the largest singularity present (Schertzer and Lovejoy, 1987; De Lima and De Lima, 2009) and it is determined by

$$\gamma_{max} = \max(K'(q)). \quad (6)$$

$K(0)$  is calculated because its value is the opposite of the codimension of the analyzed process,  $C_s$ , and characterizes weak rainfall events. At the end of the cluster analysis, each group created is a homogeneous region if it satisfies the heterogeneity test proposed by Hosking and Wallis (1997) and based on the L-Moments sample ratios. If  $t(i)$  is the  $L-C_V$  for the sample in each site of the area, one could express the regional  $L-C_V$ ,  $t_R$ , by the equation:

$$t_R = \frac{\sum_{i=1}^N n_i t^{(i)}}{\sum_{i=1}^N n_i} \quad (7)$$

Where  $n_i$  is the record length of site  $i$ . The vector  $V$  of weighted standard deviations of the at-site  $L-C_V$  is then created by the relation:

$$V = \left[ \frac{\sum_{i=1}^N n_i (t^{(i)} - t_R)^2}{\sum_{i=1}^N n_i} \right]^{\frac{1}{2}} \quad (8)$$

and the heterogeneity measure is given by,

$$H = \frac{(V - \mu_V)}{\sigma_V} \quad (9)$$

Where  $\mu_V$  and  $\sigma_V$ , are the mean and the standard deviation of  $V$ . The region is flagged as “acceptably homogeneous” if  $H < 1$ , “possibly heterogeneous” if  $1 < H < 2$  and “definitely heterogeneous” if  $H \geq 2$ .

Once homogeneous regions have been identified, a probability distribution has to be chosen for each of them. The aim is to find the one that yields accurate quantile estimates for each site. Distributions with more than three parameters are usually applied in the RFA because of the greater data availability with respect to the local one, which reduces the uncertainty in the parameters estimation. The choice between candidate distributions that could fit the sample is made with a “goodness of fit” test. In this work, the one proposed by Hosking and Wallis (1997) has been applied, as it is valid for distribution with at least three parameters. They proposed introducing the variable as an evaluation of the goodness of fit expressed as follows:

$$Z^{DIST} = \frac{(t_4^{DIST} - t_4^R + B_4)}{\sigma_4} \quad (10)$$

where  $t_4^R$  is the regional average L-Kurtosis,  $t_4^{DIST}$  is the distribution L-Kurtosis, and  $B_4$  and  $\sigma_4$  are the bias and the standard deviation of  $t_4^R$ . The Probability Distribution is considered to fit the sample if  $|Z^{DIST}| \leq 1.64$  and its goodness arises as this value gets close to zero.

The RFA's last step is the at-site quantiles estimation. In fact, once a probability distribution has been chosen, its parameters are calculated by using L-Moments ratios and the associated regional growth curve  $q(F)$  can be obtained. Then, the index flood procedure is adopted for each homogeneous region. It lets one know the quantile function  $Q_i(F)$  for the site  $i$  using the equation:

$$Q_i(F) = \mu_i q(F) \quad (11)$$

where  $\mu_i$  is the index-flood, corresponding to the sample average. So, for each return period  $Tr$ , quantiles are available and the IDF curve parameters can be estimated.

### 3. STUDY AREA AND DATA SOURCES

The study area was in the region of Umbria, central Italy, which has an extension of 8456 Km<sup>2</sup> and quite a continental climate because of the absence of any coast (Figure 1). Umbria's orography is highly complex along the north-eastern and eastern boards, where the Apennine Mountains are located, whose peaks exceed 2000 metres in altitude; the territory is typically hilly and flat in the inland valleys in the central and western areas. Because of this orographic variability, the climate is different from zone to zone. On the highland areas in the east, annual cumulative rainfall is about 900-1200 mm, while it is about 500-700 mm to the west, near Tuscany, where the influence of warm currents coming in from the Tyrrhenian Sea becomes relevant. In the same way, temperatures in the east are very low in winter and moderate in summer, while in the west they are generally higher with maxima of 30-35°C in the hottest periods, in particular in the inland valleys. Almost all the regional territory is included in the Tiber River Basin. In fact, the Tiber River crosses all the areas from north to south-west receiving water from many tributaries, which are more numerous on the hydrographic left side. Continuous hourly rainfall data series were used in this work. They were recorded by 23 rain gauges homogeneously distributed over the region and were supplied by the Umbria Region Department of Water Resources and Flood Risk Management and Umbria Region Functional Centre. The site elevations range from 109 to 941 m above mean sea level, longitude, from 11°58'09" to 13°00'50" E and latitude from 42°33'03" to 43°30' N.

Figure 1. Umbria region



### 4. RESULTS AND DISCUSSION

#### 4.1 Data quality control

No data were flagged as being erroneous in the first check of the range test. In fact, none of the extreme values exceeded lower and upper limits. After this stage, the persistence test was applied and, when it had not been satisfied, a manual inspection was necessary. Anyway, no data were discarded, showing a good efficiency of the sensors.

#### 4.2 Regional Frequency Analysis and Multifractality

Continuous hourly data series from 23 station located in the Umbria region were validated and extremes for durations of 1, 3, 6, 12 and 24 h were obtained. For each duration and each extreme data series, L-Moments values and L-Moments ratio ( $L-C_v$ ,  $L-C_s$ ,  $L-C_k$ ) were used to characterize the 23 sites. The heterogeneity test proposed by Hosking and Wallis (1997) and explained in section 2.2 was not achieved for almost any of the durations, the H values being lower than 1. Unfortunately, this did not happen for the 6 hour extreme data series. Thus a cluster analysis has been necessary to create two sub-regions, which were then submitted to the heterogeneity test. Stations were grouped by using the site characteristics obtained from the exponent function, related to the multifractal character of the continuous hourly rainfall data series. For this purpose, multifractal characterization was carried out for all the sites and the scaling of moments  $q$  determined. Figure 2 is related to Gubbio rain gauge and shows the log-log plot of the average  $q^{th}$  moment of the rainfall intensity  $\varepsilon_\lambda$  against the scale ratio  $\lambda$ . The top plot refers to the moments greater than 1 and the bottom one to the moments lower than 1. A scale behaviour of between 1 hour and 21 days can be seen, so, starting for this range, the empirical moments scaling exponent function was determined (Figure 3) and the  $\gamma_{max}$  and  $K(0)$  values obtained. Table 1 shows a synthesis of the multifractal analysis results calculated for each station.

At this point, using these parameters, the Cluster Analysis was made and two sub-regions were created: the first one composed of 14 sites and the second of 8. Only one station (Ponte Santa Maria) stayed out of them. Both the groups satisfied the heterogeneity test condition of homogeneity, this being  $H_1=0.90$  and  $H_1=0.03$ . Detailed results are shown in Table 2.

Figure 2. Log-log plot of the qth moments of hourly rainfall intensity  $\varepsilon_\lambda$  versus the scale ratio  $\lambda$  for the location of Gubbio.

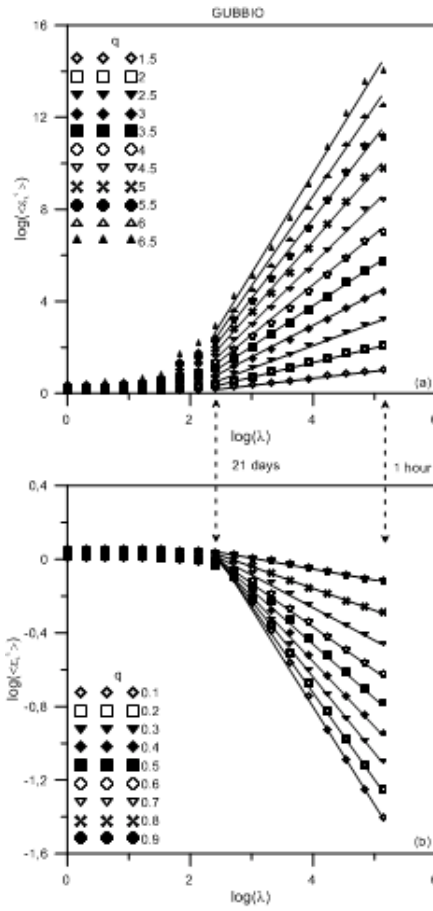
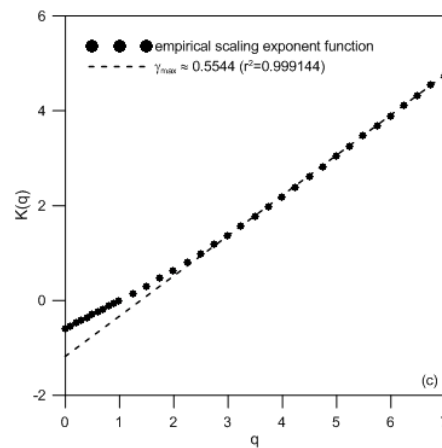


Figure 3. Empirical moments scaling exponent function  $K(q)$  for the range of scale detected at Gubbio





**Table 1. Multifractal Analysis Results for 23 sites in Umbria Region**

| STATION                | SCALE BEHAVIOUR RANGE | $\gamma_{max}$ | $K(0)$   |
|------------------------|-----------------------|----------------|----------|
| AZZANO                 | 1 hour-21 days        | 0,7982         | -0,62485 |
| BASTARDO               | 1 hour-21 days        | 0,7592         | -0,61944 |
| BASTIA UMBRA           | 1 hour-21 days        | 0,8200         | -0,62983 |
| CASACASTALDA           | 1 hour-21 days        | 0,6757         | -0,59043 |
| CASIGLIANO             | 1 hour-21 days        | 0,7736         | -0,6204  |
| CERBARA                | 1 hour-11 days        | 0,8668         | -0,63424 |
| CITTÀ DI CASTELLO      | 1 hour-21 days        | 0,7096         | -0,60499 |
| COMPIGNANO             | 1 hour-11 days        | 0,7548         | -0,65787 |
| FORSIVO                | 1 hour-11 days        | 0,8324         | -0,63951 |
| GUBBIO                 | 1 hour-21 days        | 0,8487         | -0,5932  |
| LA CIMA                | 1 hour-21 days        | 0,8022         | -0,58953 |
| MONTELOVESCO           | 1 hour-21 days        | 0,7772         | -0,61192 |
| NARNI SCALO            | 2 hour-21 days        | 0,8283         | -0,60982 |
| NOCERA UMBRA           | 1 hour-21 days        | 0,6711         | -0,5917  |
| PERUGIA S. G.          | 1 hour-21 days        | 0,7536         | -0,60431 |
| PETRELLE               | 1 hour-21 days        | 0,6949         | -0,602   |
| PONTE SANTA MARIA      | 1 hour-11 days        | 0,8981         | -0,64008 |
| PONTICELLI             | 1 hour-11 days        | 0,7327         | -0,6241  |
| RIPALVELLA             | 1 hour-21 days        | 0,7744         | -0,60361 |
| SAN BENEDETTO VECCHIO  | 1 hour-21 days        | 0,8412         | -0,60907 |
| SAN BIAGIO DELLA VALLE | 1 hour-21 days        | 0,8405         | -0,6389  |
| SAN SILVESTRO          | 2 hour-21 days        | 0,7343         | -0,59864 |
| TODI                   | 2 hour-21 days        | 0,8266         | -0,58994 |

**Table 2. Homogeneous regions obtained for the 6 hour extreme data series**

| REGION | STATIONS (22)  | $H_1$ |
|--------|--|-------|
| 1      | Bastardo, Casacastalda, Casigliano, Cerbara, Città di Castello, Compignano, Montelovesco, Nocera Umbra, Perugia Santa Giuliana, Petrelle, Ponticelli, Ripalvella, San Biagio della Valle, San Silvestro. | 0,90  |
| 2      | Azzano, Bastia Umbra, Forsivo, Gubbio, La Cima, Narni Scalo, San Benedetto, Todi   | 0,03  |

Once homogeneous areas have been created for the durations 1, 3, 12 and 24 hours, the RFA was applied considering all the 23 stations belonging to the same region; as well as the 6 hours extreme data series, two groups of sites were considered because of the presence of two homogeneous sub-regions. The probability distribution functions considered in this work were: Generalized Normal (GEN-NOR), Generalized Pareto (GEN-PAR), Generalized Extreme Value (GEV), Generalized Logistic (GEN-LOG) and Pearson Type III (PT-III). The  $Z^{DIST}$  parameter introduced in section 2.2 was calculated for each of them and each region (Table 3) and the value closest to zero indicates the one that best fits the sample data.

**Table 3.  $Z^{DIST}$  values for different probability distributions considered for each region created for the 6 hour extreme data series, those in bold indicate the best probability distribution**

| REGION | GEN-LOG | GEV   | GEN-NOR | PT-III | GEN-PAR |
|--------|---------|-------|---------|--------|---------|
| 1      | 2.89    | 0.98  | 0.74    | 0.10   | -3.21   |
| 2      | 0.29    | -0.58 | -1.09   | -1.99  | -2.81   |

When the probability distribution had been chosen, a regional growth curve was created for each region and different return period: Table 4 shows sthis for the 6 hour extreme data series.

**Table 4. Regional growth curves for the two regions obtained in the analysis of the 6 hour extreme data series and for different return periods**

| REGION | 2    | 5    | 10   | 20   | 25   | 50   | 100  | 200  | 500  | 1000 |
|--------|------|------|------|------|------|------|------|------|------|------|
| 1      | 0.95 | 1.24 | 1.42 | 1.59 | 1.64 | 1.80 | 1.95 | 2.10 | 2.29 | 2.44 |
| 2      | 0.91 | 1.21 | 1.45 | 1.71 | 1.81 | 2.14 | 2.54 | 3.01 | 3.79 | 4.52 |

This procedure has been repeated for each duration. Then, according to the Flood Index method, the extreme annual rainfall values for different return periods were calculated. Table 5 reports, as an example, their values for the Gubbio station.

**Table 5. Rainfall quantiles for return periods of 5, 10, 25, 50, 100, 200 years and different durations for the Gubbio station**

| Duration (h) | Return period (years)  |       |        |        |        |        |
|--------------|------------------------|-------|--------|--------|--------|--------|
|              | 5                      | 10    | 25     | 50     | 100    | 200    |
|              | Rainfall quantile (mm) |       |        |        |        |        |
| 1            | 32,37                  | 38,35 | 47,21  | 55,01  | 64,03  | 74,52  |
| 3            | 46,51                  | 55,24 | 66,88  | 75,96  | 85,37  | 95,17  |
| 6            | 56,48                  | 67,54 | 84,55  | 100,07 | 118,55 | 140,7  |
| 12           | 69,34                  | 81,41 | 96,99  | 108,83 | 120,84 | 133,12 |
| 24           | 84,59                  | 99,69 | 119,54 | 134,84 | 150,53 | 166,68 |

## 5. SUMMARY AND CONCLUSIONS

This work reported the Regional Frequency Analysis application to a case study in Umbria Region (Central Italy). First a data quality control was made with the use of a range and persistence test applications and a manual inspection when needed. The procedure revealed the absence of anomalous records.

After the data check, RFA was applied, following the method introduced by Hosking and Wallis (1997). Stations were grouped by a Cluster Analysis using the multifractal characteristics of the continuous hourly data series, recorded by each rain gauge, instead of other site characteristics (latitude, longitude, altitude, etc.), much more used in traditional practice; so a multifractal analysis was implemented. Specifically, the moment scale exponent function  $K(q)$  was obtained for each data series and two parameters were calculated from it:  $\gamma_{max}$  and  $K(0)$ . The first value is related to the presence of extreme events,

the second one to the low values of rainfall data. This study was applied for 23 stations and all of them presented a multifractal behaviour with a scale invariance ranging between one hour and 21 days for almost all the stations.

Once the multifractal analysis was completed, the annual extreme data series for the durations of 1, 3, 6, 12, 24 hours were submitted to the heterogeneity test proposed by Hosking and Wallis (1997). The 23 sites resulted in belonging to the same region in the case of 1, 3, 12, 24 h durations because the test was not satisfied, so in this situation the RFA was made considering the stations belonging to the same group.

In the case of data referring to the 6 hour duration, the test was satisfied and the cluster analysis was necessary to create two new sub-regions, thus verifying their homogeneity. The grouping procedure was applied by using the  $\gamma_{max}$  and  $K(0)$  multifractal parameters and the new regions, both behaving as homogeneous, were composed of 14 and 8 locations. Ponte Santa Maria station could not be added to any of them and only the Local Frequency Analysis can be applied to this data series.

The best probability distribution function was found for each region following a goodness of fit test. Then the regional growth curve was obtained, and rainfall quantiles were estimated for different return periods at each site.

The results show that multifractal characterization of rainfall data can be directly used to identify homogeneous regions in the Regional Frequency Analysis.

## 6. REFERENCES

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