COMPARATIVE ANALISYS OF NUMERICAL MODELS BASED ON FINITE ELEMENT ANALYSIS OF BLACK POPLAR PLYWOOD PANELS

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Black poplar plywood panels are a relatively new composite with a wide range of potential applications. Specially, these panels are being used for decoration and structural proposes. However, a huge number of experimental tests are required to determine the mechanical properties of these composites prior being implemented as a structural element. Here, FEM simulations arise as a promising technique to predict the behaviour of the panels under different conditions. These rapid predictions will accelerate the introduction of plywood panels in civil engineering filed without affording big expenses.

In this paper, two different approaches are tested using the commercial software ABAQUS©. In the first one, boards are modelled as a sum of individual plies with known properties while in the second one, boards are analysed as unique homogeneous entity. Both models are validated against the Kirchhoff-Love theory of plates. Results show that FEM simulations are able to accurately predict the deformation of plywood boards under bending forces. However, the more realistic homogeneous shell approach remains slightly far away from the results of the theory of plates. Therefore, future works should be focused on obtaining experimental data in order to fully understand and validate the numerical simulations.

Keywords: *Blackpoplar plywood panels; Finite element method; Theory of plates.*

ANÁLISIS COMPARATIVO DE MODELOS NUMÉRICOS BASADOS EN EL MÉTODO DE LOS ELEMENTOS FINITOS DE TABLEROS DE CONTRACHAPADO DE CHOPO

Los paneles de contrachapado de chopo son materiales compuestos con una amplia gama de potenciales aplicaciones como elementos estructurales o decorativos. Sin embargo, un gran número de ensayos son requeridos para poder determinar las propiedades mecánicas de estos compuestos antes de poder utilizarlos como elementos resistentes. En este contexto, las simulaciones FEM surgen como una técnica prometedora para predecir el comportamiento de los paneles bajo diferentes condiciones.

En este trabajo se han utilizado dos enfoques diferentes utilizando el software comercial ABAQUS ©. En el primero, los tableros se modelan como una suma de capas individuales con propiedades conocidas. Por el contrario, en el segundo se analizan los tableros como una entidad homogénea única. Ambos modelos se validan con la teoría de placas de Kirchhoff-Love. Los resultados muestran que las simulaciones FEM son capaces de predecir con precisión la deformación producida en los tableros de contrachapado a flexión. Sin embargo, el modelo de la lámina homogénea, que es más realista, se encuentra ligeramente alejado de los resultados de la teoría de placas. Por ello, los futuros trabajos deberán centrarse en la obtención de datos experimentales para poder llevar a cabo una validación más precisa de los modelos.

Palabras clave: *Tablero de contrachapado de chopo; Método de los elementos finitos; Teoría de placas*

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1. Introduction

Plywood is becoming a very popular material in civil engineering. Panels made of several materials, such as pine, spruce, birch or poplar, are being manufactured for different structural applications. In particular, Spain presents a significant production of poplar plies due to the huge quantity of existing poplar plantations (Cooper 2002).

As stated in Alia-Martinez et al. (2013), the building industry could be a future market for Spanish poplar plywood companies. Nowadays, this sector is demanding innovative techniques in order to overcome the existing economic crisis. Here, plywood panels can help to develop new market possibilities and to improve the environmental impact of buildings especially in rural areas. Besides, some studies, as the one carried out by Baldassino, Zanon, and Zanuttini (1998), have shown the adequacy and the potential of poplar plywood for structural applications by determining its mechanical properties.

Nevertheless, some major problems should be prior tackled before implementing poplar plywood at industrial level. For instance, there is a wide range of possible combinations to develop the boards. Different number of plies and several materials can be used. Therefore, too many mechanical tests are currently required to select the optimum plywood panel for each structural application.

In this context, numerical simulations based on the Finite Element Method (FEM) appear as an emerging technique, which may be able to solve this problem. With numerical simulations, the mechanical properties of each possible combination can be rapidly obtained, reducing the investment and time required to select the best panel. Consequently, FE simulations are nowadays being introduced in many I+D departments of these companies.

So far, some authors have developed FE models to analyze different properties of composite materials. Shi et al. (2012) studied the delamination process of laminated plates under a central impact. The study focused on adhesive layers, which are the most common part of failure when working with composites. Other authors, such as Ivanov and Sadowski (2009), have used the commercial software ABAQUS to simulate a compact tension test of plywood plates, gathering the elastic behavior of the material until failure. Besides, several authors have proved that the direction of the fibbers has a direct influence on the displacements in the plate under bending forces (Kljak, Brezovic, and Antonovic 2009, Sliseris and Rocens 2013).

However, despite of the existing works, the simulation of plywood panels using FEM theory is still in its infancy. These studies focus only in a small number of materials. Simulating a new material requires a significant effort. Results from previous studies cannot be directly extrapolated due to the particularities of each compound. Moreover, some of these studies do not compare they results against experimental data or against any analytic theory.

Therefore, our goal in this work is to develop FE models of poplar plywood boards and to compare the obtained results with the analytic equations of Kirchhoff-Love theory of plates. For this purpose, the displacements of a simply supported poplar plywood plate submitted to a distributed bending load will be analyzed.

2. Problem description

Two main challenges appear when simulating plywood panels.

First, modeling composite materials is not a naïve task. Two different approaches can be used to study plywood. On the one hand, boards can be analyzed as a sum of individual plies with known properties (composite shell approach). On the other hand the whole board can be considered as a homogeneous material where new experiments are required to obtain the global behavior of the sum of different plies (homogeneous shell approach). In both approaches, it is crucial to perform accurately the characterization experiments. For instance, del Coz Díaz et al. (2008) proved that FEM results strongly depend on the quality of the material characterization simulating agglomerated boards. However, these experiments are very cost demanding for the enterprises, which rarely afford these expenses. In this situation, some authors have developed standards to predict the properties of the compound, the plywood board, based on the properties of the individual material, the wood ply. For instance, Arriaga-Martitegui, Peraza-Sánchez, and García-Esteban (2008) verified the suitability of ENV-14272 to obtain mechanical properties of pine plywood.

Second, FE simulations have to be validated against experimental data in order to guarantee the reliability of the results obtained. However, as aforementioned, the characterization experiments are rarely carried out. Another option here is to compare FEM results against analytic theories. Based on the existing works, the most appropriate theory for modeling plywood panels under distributed bending forces is the Kirchhoff-Love theory of plates (Stürzenbecher, Hofstetter, and Eberhardsteiner 2010).

Consequently, the goal of this paper is to verify whether the results from Kirchhoff-Love theory of plates can be used to evaluate the quality of FE simulations. This approach was already successfully performed by Rouzegar and Mirzaei (2013) by using isotropic materials, such as steel. The challenge of this study is to assess the same methodology with an orthotropic material, poplar wood.

3. Case of study and methodology

3.1. Material description

Plywood panels studied in this paper are made of poplar lumber. These panels consist of 9 plies and a total nominal thickness of 19,3 mm. The different layers were glued using urea-formaldehyde adhesive. Table 1 depicts poplar properties. The module parallel to the grain direction was obtained using the standards shown in AENOR (2003). Equations described in Kretschmann (2010) were used to calculate both the perpendicular elasticity module and the shear modules.

Table 1. Poplar properties

E //	E	**v**	G_{13}	G_{12}	G_{32}
7500	322	0.3	562	518	83

Properties of plywood panels differ from the properties of poplar plies alone when the boards are studied as a composite. The orientation of the plies is changed in order to improve the final characteristics of the boards. Particularly, the board studied presents the following layup: 0º, 90º, 0º, 90º, 0º, 90º, 0º, 90º, 0º. This configuration seeks to obtain boards that present a similar behavior in both principal directions. The final properties of the board, obtained following the tests described in AENOR (2006), are showed in Table 2.

Table 2. Board properties

E //	E	**v**	G_{13}	G_{12}	G_{32}
3200	4600	0.3	300	300	300

3.2. FEM model

Numerical simulations are based on traditional differential elasticity equations. Elasticity equations, which are easily derived for a three dimensional problems, are not solvable by analytic methods. FEM theory tackles this problem by dividing the domain of interest into a mesh of finite elements. In these elements, interpolating polynomials are used to approximate the solution of the system of governing equations. After several iterations, the final solution is obtained.

The elasticity theory is based on the study of the strain and stress matrices:

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \quad (4) \qquad \varepsilon = \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \varepsilon_{zz} \end{bmatrix} \quad (5)$$

Considering the external stress and the bearing conditions as known parameters, the unknowns of those matrices can be solved.

The deformation of a single point is defined using a vector u, which has as much components as deformations appear in the domain. When the problem is three-dimensional the structure of the vector is:

$$u = \begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix} \quad (6)$$

Considering any finite element, the strain field inside the element is approximated using an interpolation hypothesis such as the weighted average of the strains in each of the n nodes of the element. The weighting factors in the interpolation functions are as follows:

$$u = \sum N_t \cdot U_t \, ; v = \sum N_t \cdot V_t \, ; w = \sum N_t \cdot W_t \qquad (7)$$

This interpolation process can be expressed in matrix form:

$$u = N \cdot \delta^e \quad (8)$$

Where δ^e is the vector with all the nodal distributions of the element.

$$\delta^e = \{U_1 V_1 W_1 U_2 V_2 W_2 \dots U_n V_n W_n\}^T \quad (9)$$

The interpolation matrix has always 3 rows and as many columns as degrees of freedom exist between all the nodes of the element. The structure of this matrix is shown in equation 10.

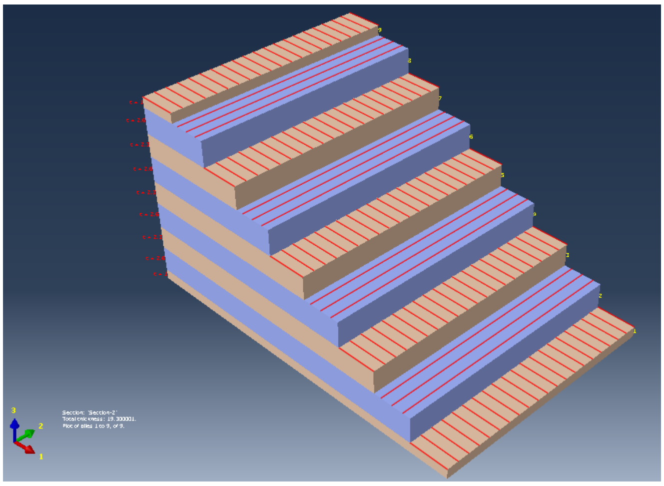
$$N = \begin{bmatrix} N_1 & 0 & 0 & \cdots & \cdots & N_n & 0 & 0 \\ 0 & N_2 & 0 & \cdots & \cdots & 0 & N_n & 0 \\ 0 & 0 & N_3 & \cdots & \cdots & 0 & 0 & N_n \end{bmatrix} \quad (10)$$

In this study two different FEM models have been developed using the commercial software ABAQUS.

FEM Model 1: composite shell

In this first model, the board is defined as a conventional shell with a composite section. Conventional shell elements discretize a body by defining the geometry at a reference surface. In this case the thickness is defined through the section property definition. Properties used are the poplar ones (table 1) and the composite layup was made using the composite module included in ABAQUS. The layup is showed in Figure 1.

Figure 1. Composite Layup



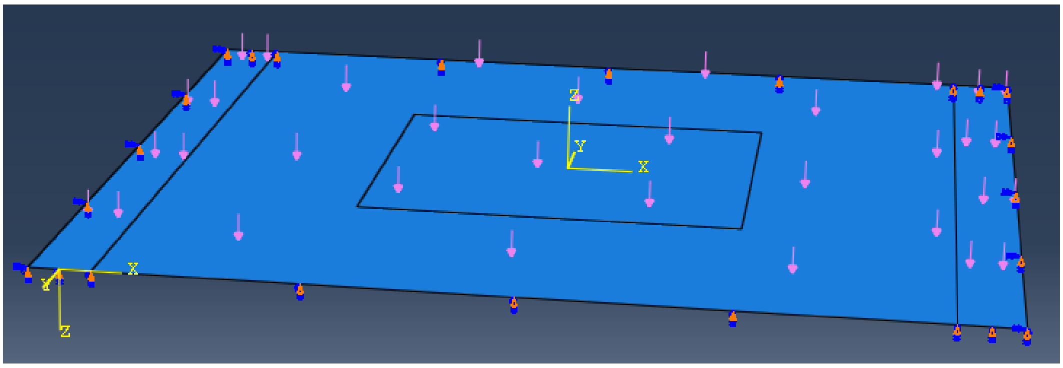
FEM Model 2: homogeneous shell

In this second model, the panel is also developed as a conventional shell. However, instead of defining all the plies in the panes, the section is considered to be homogeneous. The board is studied as a unique entity. Board properties of table 2 are used.

In both cases the elements used to form the mesh were S4R with an approximate global size of 40. S4R is a 4-node, quadrilateral, stress/displacement shell element with reduced integration and a large-strain formulation (ABAQUS 2004). Five different distributed loads were applied on them (from 0.001 N/mm2 to 0.005 N/mm2). The steps used are static general. When using static analysis inertia effects are neglected but nonlinearities can be calculated. This is an important characteristic for our study due to our material can have a nonlinear behavior. Newton's method is used to solve the nonlinear equilibrium equations.

Figure 2 shows the loading and bearing conditions.

Figure 2. Loading and bearing conditions



3.3. Kirchhoff-Love theory of plates

The Kirchhoff-Love theory of plates is used to validate the FEM model results. This theory is based in the following assumptions:

- Straight lines perpendicular to the mid-plane before the deformation remain perpendicular after deformation.
- Longitudinal and transversal dimensions of the plate must be 10 times bigger than its thickness.

- Displacements obtained are significantly smaller than the thickness.

Besides, in this study, layers will be considered as perfect bonds. This means that there is no space between them before or after the deformation.

Navier solution is used to solve the system of governing equations. The resulting equations are as follows (Blazquez-Gamez 2004):

$$\nabla_w{}^4 = \frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \cdot \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} \qquad (11)$$

Sides of the plate are defined by $x = 0$, $x = a$, $y = 0$ and $y = b$ and the boundary conditions that must be satisfied are:

1. $w = 0$.
2. $x = 0$ and $x = a$ is $\frac{\partial^2 w}{\partial x^2} = 0$.
3. $\frac{\partial^2 w}{\partial y^2} = 0$ in $y = 0$ and $y = a$.

The real plate is supposed as part of another fictitious plate four times greater and it is submitted to an odd charge: $p(x, y) = -p(-x, y) = -p(x, -y) = p(-x, -y)$. This conditions implies that movements can be also considered as an odd function, $w(x, y) = -w(-x, y) = -w(x, -y) = w(-x, -y)$. The period in the x direction is $T = 2 \cdot a$, and $T' = 2 \cdot b$ in the y direction. In order to solve the problem using Fourier series, $w = \frac{\pi}{a}$, $w' = \frac{\pi}{b}$ is imposed and it is considered that the charge can be decomposed as shown in equation 12.

$$q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mn} \cdot \sin \frac{m \cdot \pi \cdot x}{a} \cdot \sin \frac{n \cdot \pi \cdot y}{b} \qquad (12)$$

The deflection function is showed in equation 13.

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{16 \cdot q_0 \cdot \sin \frac{m \cdot \pi \cdot x}{a} \cdot \sin \frac{n \cdot \pi \cdot y}{b}}{\pi^6 \cdot m \cdot n \cdot \left[D_{11} \cdot \left(\frac{m}{a}\right)^4 + 2 \cdot (D_{12} + 2 \cdot D_{66}) \cdot \left(\frac{m}{a}\right)^2 \cdot \left(\frac{n}{b}\right)^2 + D_{22} \cdot \left(\frac{n}{b}\right)^4 \right]} \qquad (13)$$

For m and n = 1,3,5,7,9 …

The resulting equations are solved using the iterative Newton method in the commercial software Matlab.

4. Results and discussion

Table 3 shows displacements obtained each different model.

Table 3. Displacements (mm)

Load (N/mm2)	Theory of plates	FEM Model 1	FEM Model 2
0.001	-0.3496	-0.3604	-0.3127
0.002	-0.6993	-0.7208	-0.6254
0.003	-1.0489	-1.0811	-0.9381
0.004	-1.3986	-1.4415	-1.2508
0.004	-1.7482	-1.8019	-1.5635

Note: FEM Model 1: Shell Composite Section, FEM Model 2: Shell Homogeneous Section

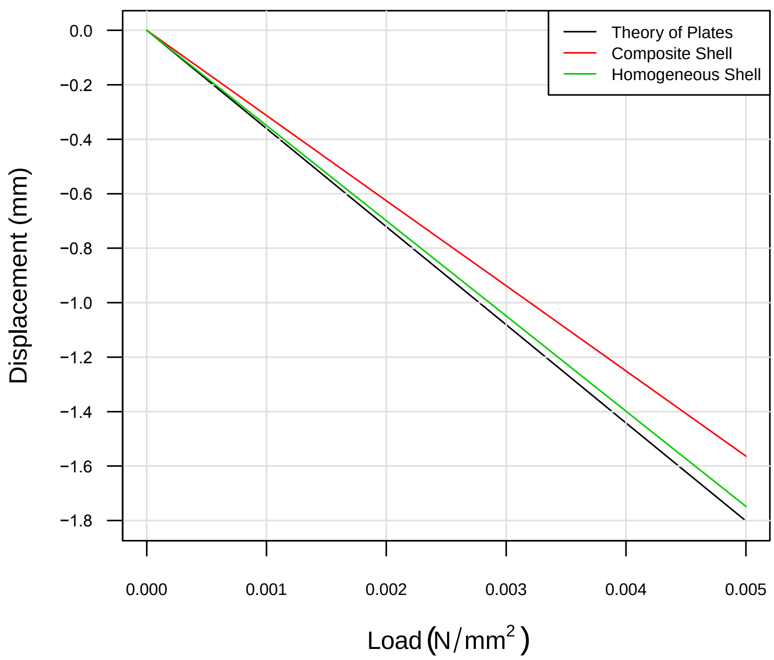
Considering the results obtain with the Kirchhoff-Love theory of plates as the reference solution, the errors for both FEM models are computed. These errors are shown in Table 4.

Table 4. Error

Load (N/mm2)	FEM Model 1 (%)	Error Model 2 (%)
0.001	3.082380	10.55664
0.002	3.067639	10.56943
0.003	3.072743	10.56516
0.004	3.067353	10.56914
0.004	3.070587	10.56687

Displacements are plotted in figure 3 to see their distribution through the width of the board. It is shown how both models and the theory of plates follow linear distributions. As the displacements obtained are quite small, the tendency observed agrees with theory of elasticity.

Figure 3. Displacements



Both, table 4 and figure 3, show that the option that seems closer to the results obtained with the Kirchhoff-Love theory of plates is the composite shell approach implemented in FEM Model 1. The reason is that both, FEM Model 1 and theory of plates, consider that joints between the layers are perfect bonds. However, in FEM Model 2, the properties used in the homogeneous section were obtained from a real board were those bonds are not perfect.

This means that the composite shell approach is the one that better approximates the theory of plates equations. However, the homogeneous shell approach, where the material properties are based on experiments performed through the whole plywood panel, should better approximate the reality because it does not make the perfect bonds assumption. More experimental data is required to confirm this idea.

5. Conclusions

In the previous study, the adequacy of FEM simulations to model the behaviour of poplar plywood boards under bending forces has been evaluated. Two different approaches were implemented, where the first one (FEM Model 1) modelled the material as a composite made of several plies while the second one (FEM Model 2) described the board as a homogeneous material. Both approaches yield acceptable results compared to the traditional Kirchhoff-Love theory of plates. This implies that, when no experimental data is available, the predictions of Kirchhoff-Love theory can be used to evaluate the accuracy of FEM simulations.

Nevertheless, in the homogeneous shell approach where the assumption of perfect bonds is not imposed, results slightly differ from the solution of theory of plates. This entails that future works should be focus on obtaining experimental data in order to gather all the existing irregularities and fully validate the adequacy of FEM simulations.

Finally, this study proves that both, Kirchhoff-Love theory of plates and FEM simulations, can be used to obtain approximate predictions of the displacements of poplar plywood plates. Therefore, plywood enterprises could reduce the amount of mechanical tests required to sell their products to the building industry.

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