01-002

A NEW EVM-DRIVEN AND RISK-AWARE INDEX RULE PROPOSAL FOR PROJECT SCHEDULING AND CONTROL USING WHITTLE APPROACH

Taboada, Ianire [1]; Toledo Gandarias, Nerea [1]

[1] UPV/EHU

This paper deals with the problem of scheduling project tasks under uncertainty and with limited resources. In this context and under an EVM-driven project control, we formulate a risk-aware scheduling model as a Markov Decision Process, and then we derive a simple index rule using Whittle approach. By means of this method we obtain a metric per-task state that measures the dynamic service priority of a single task; it consists in allocating resources to the tasks with the current highest productivity of using the resource. The designed scheduling solution is evaluated in a realistic scenario. The results indicate that the proposed index rule improves project performance.

Keywords: project scheduling and control; Earned Value management; project risk management; Whittle index; Markov Decision Process

PROPUESTA PARA LA PLANIFICACIÓN Y CONTROL DE PROYECTOS BASADA EN EVM Y CON GESTIÓN DE RIESGOS USANDO WHITTLE

Este artículo trata el problema de planificación de tareas de proyectos con incertidumbre y recursos limitados. En este contexto y bajo un seguimiento del proyecto basado en EVM, formulamos un modelo de planificación con control de riesgos como un Proceso de Decisión de Markov, y después derivamos una regla de índice simple usando el enfoque de Whittle. Mediante este método se obtiene una métrica por estado de tarea que mide la prioridad dinámica de servicio de una única tarea; consiste en asignar recursos a las tareas con la actual productividad de uso de recurso más alta. La solución de planificación diseñada se evalúa en un escenario realista. Los resultados indican que la regla de índice propuesta mejora el rendimiento del proyecto.

Palabras clave: planificación y control de proyectos; gestión del Valor Ganado; gestión de riesgos del proyecto; índice de Whittle; proceso de Decisión de Markov

Correspondencia: Ianire Taboada ianire.taboada@ehu.eus

1. Introduction

Project scheduling involves finding a start and finish time for all the project activities, considering a set of limitations such as precedence relations, temporary restrictions and resource constraints, while a given scheduling objective is optimized. Classical scheduling methods based on graph theory such as critical path method (CPM) (Kelley Jr & Walker, 1959) and program evaluation and review technique (PERT) (Malcolm, Roseboom, Clark, & Fazar, 1959) allow mechanically calculating the start and finish times of project tasks while optimizing project duration. However, these methods consider that all the resources required by the tasks would be available at the time of their execution, which is an unrealistic assumption in practice. Moreover, they do not take into account the uncertainty caused by project risks either. Further, it is demonstrated that the resource-constrained project scheduling project is a NP-hard problem (Blazewicz, Lenstra, & Kan, 1983), so its optimal solution is unfeasible.

Besides, project control consists in the comparison of a baseline (i.e., the initial project plan) with the actual results of the project to identify deviations and activate corrective early actions if needed. Earned Value Management (EVM) is a widely used project management technique for project performance monitoring (Anbari, 2003). EVM integrates schedule, cost and scope control under the same framework, and it provides performance indicators which allow project managers to detect delays and over-costs. These indicators are schedule variance (SV = EV - PV) and cost variance (CV = EV - AC), where PV is the planned value or the budgeted cost of the work scheduled, AC the actual cost of the work actually performed and EV the earned value or the planned cost of the work actually completed. A negative variance shows that the project is behind schedule or exceeding the planned budget. Therefore, by monitoring the evolution of these parameters during the project life cycle, project managers can detect deviations from the plan, so that they can take early corrective actions. In this paper, as a corrective action, we adopt the approach of project tasks re-assignment according to a well-performing scheduling policy.

Scheduling processes can be formulated as an optimization problem where the objective is to optimize a given parameter, such as project duration. The scheduler can be viewed as a controller where the control action comprises of determining the tasks to be assigned to each resource. Thus, these processes can be formulated as control problems. Stochastic and dynamic resource allocation problems are naturally modelled in the framework of Markov Decision Processes (MDP) (Puterman, 1990). Nonetheless, the main drawback of using these markovian control processes for solving real optimization problems is that most practical problems require unachievable memory and processing requirements to be solved. To avoid this, it is more adequate to formulate the resource management optimization problems as a MDP, and then, obtain a simple suboptimal or even optimal heuristics as the solution to the complex and intractable problems considered.

The approach of Whittle (Niño-Mora, 2007; Whittle, 1988) allows to derive nearly-optimal heuristic scheduling rules by computing an index policy that consists in allocating resources to the jobs with the highest productivity of using the resource. It is worth mentioning that Whittle's work was formulated in the MDP framework, and then, a simple and suboptimal index rule was achieved. Whittle method has been broadly used during the last years for proposing simple and well-performing schedulers in different areas (Aalto, Lassila, & Taboada, 2019; Villar, Bowden, & Wason, 2015). Nevertheless, to the best of our knowledge, there is no scheduling heuristic rule proposal that applies Whittle index rule theory in project management. Moreover, the MDP modelling of the EVM-driven risk-aware scheduling problem in that field is something novel.

Hence, the main contribution of this paper is twofold:

- Firstly, the EVM-driven risk-aware scheduling problem is formulated as a suitable MDP model. This general formulation will result useful for researchers interested in project scheduling field as the basis for dealing with different or/and more complex project scheduling optimization problems in an analytical fashion.
- Secondly, in order to solve the aforementioned risk-dependant MDP formulation, we focus on designing a novel and simple risk-aware heuristic rule based on Whittle approach that gives satisfactory performance in terms of project duration.

The rest of the paper is structured as follows. First of all, the EVM-driven risk-aware project scheduling problem is described in Section 2. Then, in Section 3 we formulate the problem as a MDP. In Section 4 an index rule based on Whittle approach is derived. Then, its performance is analysed in Section 5. Finally, Section 6 gathers the main conclusions of this work.

2. Problem description

We analyse the problem of scheduling project tasks with limited shared resources and uncertainty, where the objective is to minimize project duration. The preliminary project schedule is created following CPM. By default project tasks are assigned to the initially assigned resources. However, in each project control point the scheduler is aware of EVM indicators (CV, SV). When based on these parameters a resource re-allocation is considered as a corrective action, in that control slot the scheduler makes decisions in order to choose which resource is assigned to each ongoing task.

Each task i is characterized by its random size, X_i, this is, the total amount of days to be executed. The task size follows an exponential distribution, with expectation $E[X_i]$. The scheduling algorithm used will determine the time needed to perform the total task execution. Each task has an execution speed, $s_{i,j}$, per each j resource available, which represents the ratio of the daily task execution (i. e., $s=2$ means that the task is executed twice faster in that resource). We denominate the cost per day of executing a task i using a resource j as $C_{i,j}$.

The risk of a task i in a resource j can take $N_{i,j,n}$ conditions from a finite set $\mathbb{N}:=\{1, 2,..., N_{i,j,n}\}$. These risk conditions are associated to different impact factors ($f_{i,j,n}$), where $f_{i,j,1} \leq f_{i,j,2} \leq ... \leq f_{i,j,N_{i,j,n}}$, which affect the speed of using resource j. So, the resulting execution rate of the resource j for task i, $r_{i,j}$, is equal to the execution speed $s_{i,j}$ multiplied by that impact factor related to the risk. The risk condition of a task i evolves randomly and independently of other tasks. We denote the probability of being in risk state n by $q_{i,j,n}$, having $\sum_{n \in \mathbb{N}} q_{i,j,n} = 1$.

In the next section, the presented scheduling problem is modelled by a MDP.

3. MDP formulation

A MDP is a powerful mathematical framework which is used for studying a wide range of optimization problems. In this manner, the scheduling problem described in Section 2 can be formulated as a MDP.

We could define a MDP as a discrete time stochastic control process. At each time slot, the process is in some state s, and a decision maker may choose any action, a, that is available in that state, getting a corresponding reward R_s^a. Consequently, the process moves randomly to a new state s' with probability $p^a(s,s')$.

A control action is a decision made at a given time, whereas a policy is a rule for selecting actions as a function of time and information available to the controller. This way, the decision maker faces a trade-off between exploitation and exploration. On the one hand, we mean by exploitation aiming at getting the highest immediate rewards at present. On the other hand, we refer to exploration by obtaining a possibly higher reward in the future, learning about the system and receiving possibly even higher rewards later.

We are interested in applying a MDP in order to decide in each EVM-driven control point which resource is assigned to each project task under execution, with the final objective of minimizing project duration. This way, the evolution of the MDP will determine the scheduling policy aimed at minimizing project duration by maximizing the aggregate reward; this is, in the sense that in each decision slot an action- and state-dependent reward is earned per-task, which computes in the final aggregate reward.

Next, we describe the elements of the proposed risk-aware scheduling MDP model. To do so, the corresponding action space, states, state transition probabilities, rewards and work are defined in a per-task and per-resource basis.

Every task *i* can be assigned or not to a resource *j*. If the action for a task *i* in a resource j, a_{ij}, is 1, this task is selected for using resource *j*; otherwise, $a_{ij}=0$.

Once a task is ready to be scheduled, states in the per-task and per-resource space are classified in two groups:

- Scheduling states (n): Task *i* is a candidate to be scheduled in resource *j* with risk state *n* under action a_{ij}.
- Final state *: This state represents that task execution is finished.

The one-slot state-transition probabilities from state *s* to *s'* under action a_{ij} for task *i* in resource *j*, $p_{i,j}^{a_{ij}}(s,s')$, are defined as follows:

$$p_{i,j}^{0}(n,m) = q_{i,j,m} \tag{1}$$

$$p_{i,j}^{1}(n,m) = q_{i,j,m}\left(1-\mu_{i,j,n}\right) \tag{2}$$

$$p_{i,j}^{1}(n,*) = \mu_{i,j,n} \tag{3}$$

$$p_{i,j}^{0}(*,*) = 1 \tag{4}$$

Where $\mu_{i,j,n}$ is the task completion probability, which is equal to $r_{i,j}/E[X_i]$.

Regarding the definition of rewards, $R_{i,j,s}^{a_{ij}}$ is the expected one-slot reward earned by task *i* in resource *j* at state *s* if action a_{ij} is decided at the beginning of a slot; it is defined as the expected cost of remaining in the system as:

$$R_{i,j,n}^{0} = -k_{i,j} \tag{5}$$

$$R_{i,j,n}^{1} = -k_{i,j}.(1-\mu_{i,j,n}) \tag{6}$$

$$R_{i,j,*}^{a} = 0 \tag{7}$$

Where $k_{i,j}$ is the holding cost per slot.

Finally, the expected one-slot capacity consumption or work required by task *i* in resource *j* at state *s* if action a_{ij} is decided at the beginning of a slot is:

$$W_{i,j,s}^{0} = 0 \tag{8}$$

$$W_{i,j,s}^{1} = C_{i,j} \tag{9}$$

It is known that optimally solving the joint MDP problem associated with the single-task and single-resource model presented is unfeasible in general (Whittle, 1988). For that purpose, in the next section we will propose a heuristic based on Whittle method.

4. Index rule proposal

In this section a novel index rule is proposed for the model in Section 3. In such paradigm, the philosophy of an index rule consists in calculating for each task-resource a certain price called index, such that in each re-assignment slot the scheduler chooses the task-resource combination with the highest index value. Here we provide an index rule type approximate solution to the problem presented in the previous section based on Whittle method.

As defined in (Niño-Mora, 2007) the Whittle index represents the rate between the marginal reward and the marginal work, where the marginal reward (work) is the difference of the expected reward earned (work required) by being scheduled and not being scheduled at initial state (*s*) and employing policy *F* afterwards. From now on we omit task-resource labels *i,j*. Thus, we can formally write the Whittle index, v_n, for our problem as:

$$v_n = \frac{\mathbb{R}_{\mathrm{n}}^{<1,F>} - \mathbb{R}_{\mathrm{n}}^{<0,F>}}{\mathbb{W}_{\mathrm{n}}^{<1,F>} - \mathbb{W}_{\mathrm{n}}^{<0,F>}} \tag{10}$$

From the definition of rewards and works in the previous section we have:

$$\mathbb{R}_n^F = \begin{cases} -k(1-\mu_n) + \beta(1-\mu_n)\sum_{m\in\mathbb{N}} q_m \mathbb{R}_m^F & \mathrm{n} \in F \\ -k + \beta \sum_{m\in\mathbb{N}} q_m \mathbb{R}_m^F & \mathrm{n} \notin F \end{cases} \tag{11}$$

$$\mathbb{W}_n^F = \begin{cases} C + \beta(1-\mu_n)\sum_{m\in\mathbb{N}} q_m \mathbb{W}_m^F & \mathrm{n} \in F \\ \beta \sum_{m\in\mathbb{N}} q_m \mathbb{W}_m^F & \mathrm{n} \notin F \end{cases} \tag{12}$$

Where $0 \le \beta \le 1$ is a given discount factor in order to exponentially decrease rewards and costs over time.

So by substituting (11) and (12) in (10), for any state *n* and under any *F* we obtain:

$$v_n = \frac{k\mu_n - \beta\mu_n \sum_{m\in\mathbb{N}} q_m \mathbb{R}_m^F}{C - \beta\mu_n \sum_{m\in\mathbb{N}} q_m \mathbb{W}_m^F} \tag{13}$$

Nevertheless, so as to achieve a closed-form characterization of the Whittle index (13) in a state *n* it is necessary to determine the optimal policy *F* of the future states that have influence on the index computation. We can assume that a state with a better risk condition will be better than a state with a worse risk condition. In this way, we can suppose that future states where $m>n$ are active (selected for being assigned, $m \in F$), whereas $m<=n$ passive (the opposite).

In this manner, referring to reward elements, using (11):

$$R_a = \sum_{m\in\mathbb{N}} q_m \mathbb{R}_m^F = \left(1 - \sum_{m>n} q_m\right)(-k + \beta R_a) + \sum_{m>n} q_m \left[-k(1-\mu_m) + \beta(1-\mu_m)R_a\right] \tag{14}$$

And by isolating R_a in (14):

$$R_a = \frac{-k(1-\sum_{m>n} q_m\mu_m)}{1-\beta+\beta\sum_{m>n} q_m\mu_m} \quad (15)$$

Analogously, for work elements, using (12):

$$W_a = \frac{C\sum_{m>n} q_m}{1-\beta+\beta\sum_{m>n} q_m\mu_m} \quad (16)$$

Substituting (15) and (16) in (13), and simplifying:

$$v_n = \frac{\mathrm{k}\mu_\mathrm{n}}{\mathrm{C}(1-\beta+\beta\sum_{\mathrm{m}>n} \mathrm{q_m}(\mu_\mathrm{m}-\mu_\mathrm{n}))} \quad (17)$$

Aimed at minimizing project delay, we are interested in the undiscounted case, *β=1*, and *k=1*. Thus, the Whittle index for our problem is given by:

$$v_n = \frac{\mu_\mathrm{n}}{\mathrm{C}(\sum_{\mathrm{m}>n} \mathrm{q_m}(\mu_\mathrm{m}-\mu_\mathrm{n}))} \quad (18)$$

As can be observed in expression (18), the obtained index is computationally tractable. This allocation strategy is a risk-aware and cost-dependant size-based policy, which depends on risk impact and probabilities, mean task duration, and resource cost and rate. The index value equals the ratio between the instantaneous completion probability and the expected potential improvement of the instantaneous completion probability multiplied by the cost of using the resource. Further, in the best risk condition the index is infinite, since the summation in the denominator is null.

$$v_N = \lim_{\beta\to 1} \frac{\mu_\mathrm{n}}{\mathrm{C}(1-\beta)} = \infty \quad (19)$$

We will call the Whittle index obtained here *Risk-aware Cost-dependant Potential Improvement* (RCPI). In this way, using the RCPI index, we propose that the RCPI index rule consists in: at every decision slot *t*,

- assigning a task in its lowest risk condition with the highest value of $\mu_\mathrm{n}/\mathrm{C}$;
- if there is no task in its lowest risk condition, assigning the task with the highest value of (18).

Hence, the RCPI index rule can be easily implemented. Moreover, in the next section, we will show that the performance of this Whittle index heuristic is adequate.

5. Results

In this section we analyse the performance of the RCPI index rule presented in the previous section. To that end, we compare its achieved final project schedule, project duration and project cost with an ideal case and a default policy. In the ideal case we obtain the project schedule with CPM, assuming that the deterministic task duration is equal to an exponential mean size and risk free. The default scheduling discipline consists in assigning to each task that is starting its execution the fastest available resource (e.g., the most suitable person to perform that activity); in case of ties the resource with lowest cost is chosen. The RCPI scheduler is EVM-driven every month (30 days): if CV or SV is lower than zero, RCPI is performed; otherwise, no re-assignment is done and tasks are allocated according to the default policy when they are ready for being executed (their predecessors are completed).

The AON network of this case study is represented in Figure 1. In this network tasks are modelled as exponential, whose means are shown in Table 1. For this setting we have five resources with *s=1* and *C=1* for all the tasks. Starting tasks (1-3 and 7) are assigned to resources 1-4 initially.

Figure 1: AON network (Acebes, Pajares, Galán, & López-Paredes, 2014)

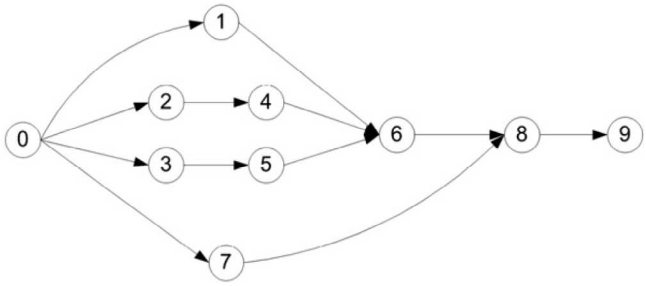

Table 1: Duration of tasks

Task Id	1	2	3	4	5	6	7	8
E[X]	150	30	90	120	60	90	240	90

Furthermore, we assume that all the tasks are risk-free, except task 7. In reference to the risk model of task 7, we consider three risk levels (high, medium, low) with $f_1=0.25$, $f_2=0.75$, $f_3=1$ and $q_1=0.3$, $q_2=0.5$, $q_3=0.2$ for resources 4 and 5 after 100 days.

We have implemented the whole project control and scheduling environment in MATLAB. We have carried out 1000 rounds of simulations for each setting considered. These rounds differ in the randomly generated task durations and risks.

Next we describe the results obtained in the analysed scenarios. Figure 2 provides final project schedules, where the effect of risks is reflected in the delay of task 7 and consequently in its successor task 8. Nevertheless, this delay is notably higher when using the default policy. Moreover, Table 2 collects project performance results both in duration and cost terms. Regarding project duration, the project delay is a 21% higher respect to the ideal case with the default discipline, whereas using RCPI only worsens a 2.4%. Concerning the cost aspect, RCPI is not too affected, only an increase of 0.8%, contrarily to the default discipline which grows in an 8%; this is due to the delay caused in task 7.

Figure 2: Final schedule comparison

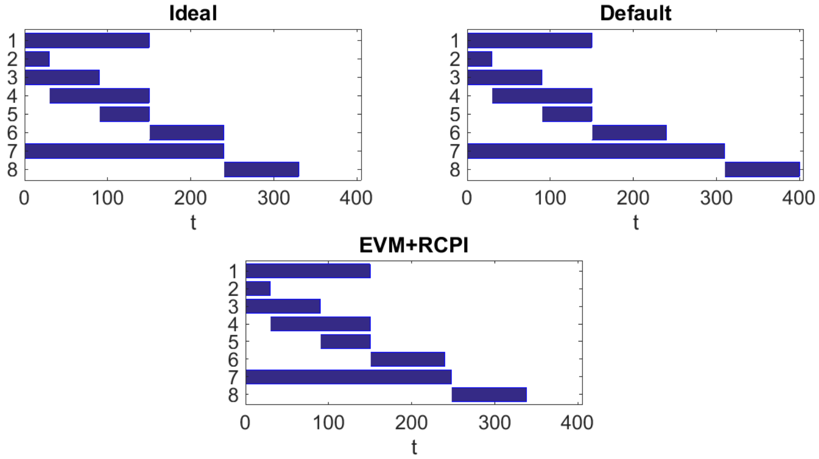

Table 2: Project performance

	Duration	Cost
Ideal	330	870
Default	400	940
EVM+RCPI	338	878

6. Conclusions

This paper constitutes an analytical advance towards incorporating risk-awareness in the design of project scheduling algorithms for controlling projects under uncertainty. The proposal presented is generic enough to be useful for project scheduling and control in different kinds of projects.

The first contribution is related to the modelling of the risk-aware project delay minimization problem when scheduling project tasks with random exponential duration under limited resources, which is suitably formulated as a MDP in Section 3. Nonetheless, the achieved model cannot be optimally solved, which motivates us to infer an approximate solution. As main contribution, in Section 4 we apply Whittle approach to derive a simple heuristic index rule as a suboptimal solution to the addressed problem. As outcome we obtain a risk-aware and cost-dependant size-based index rule that prioritizes tasks with the highest ratio between the instantaneous completion probability and the expected potential improvement of the instantaneous completion probability multiplied by the cost of using the resource. The achieved computationally tractable closed-form expression makes possible the lightweight implementation of the proposed index rule in any scheduling logic.

As discussed in Section 5, simulation results show that our novel EVM-driven index rule proposal behaves satisfactorily under the influence of risks. Numeral results indicate a significant performance improvement not only for project duration, but also for project cost. Therefore, practically, this new well-performing EVM-based risk-aware re-assignment policy will be very useful for project managers in order to guarantee adequate project performance levels.

In future research, the findings of this work could be employed as the starting point of solving more complex risk-aware project task scheduling problems, such as considering other stochastic evolution related to the risk model or task duration. Another issue that remains as future topic is not restricting our study to a single project only; that is, extending our approach to resource-constrained multi-project environments (Villafáñez, Poza, López-Paredes, Pajares, & del Olmo, 2019). Besides, even though simulation results illustrate that the proposed scheduler gives satisfactory performance, it would be interesting to show its validity in emerging projects.

7. References

Aalto, S., Lassila, P., & Taboada, I. (2019). Whittle index approach to opportunistic scheduling with partial channel information. *Performance Evaluation, 136*, 102052.

Acebes, F., Pajares, J., Galán, J. M., & López-Paredes, A. (2014). A new approach for project control under uncertainty. Going back to the basics. *International Journal of Project Management, 32*(3), 423-434.

Anbari, F. T. (2003). Earned value project management method and extensions. *Project management journal, 34*(4), 12-23.

Blazewicz, J., Lenstra, J. K., & Kan, A. R. (1983). Scheduling subject to resource constraints: classification and complexity. *Discrete applied mathematics, 5*(1), 11-24.

Kelley Jr, J. E., & Walker, M. R. (1959). *Critical-path planning and scheduling.* Paper presented at the Papers presented at the December 1-3, 1959, eastern joint IRE-AIEE-ACM computer conference.

Malcolm, D. G., Roseboom, J. H., Clark, C. E., & Fazar, W. (1959). Application of a technique for research and development program evaluation. *Operations research, 7*(5), 646-669.

Niño-Mora, J. (2007). Dynamic priority allocation via restless bandit marginal productivity indices. *Top, 15*(2), 161-198.

Puterman, M. L. (1990). Markov decision processes. *Handbooks in operations research and management science, 2*, 331-434.

Villafáñez, F., Poza, D., López-Paredes, A., Pajares, J., & del Olmo, R. (2019). A generic heuristic for multi-project scheduling problems with global and local resource constraints (RCMPSP). *Soft Computing, 23*(10), 3465-3479.

Villar, S. S., Bowden, J., & Wason, J. (2015). Multi-armed bandit models for the optimal design of clinical trials: benefits and challenges. *Statistical science: a review journal of the Institute of Mathematical Statistics, 30*(2), 199.

Whittle, P. (1988). Restless bandits: Activity allocation in a changing world. *Journal of applied probability, 25*(A), 287-298.

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