BUILDING A BID TENDER FORECASTING MODEL: SCORING AND POSITION PROBABILITY GRAPHS

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Abstract

The strategy of each bidder will differ depending on the ESF selected and the weight of the overall proposal scoring.

In this paper a new Bid Tender Forecasting Model (BTFM) has been developed. The various mathematical relationships and density distributions that describe the main SPs and FPs, and the representation of tendering data by means of iSCGs, enable the generation of two new types of graphs that can be very useful for bidders to define a competitive strategy.

Keywords: *bid; tender; auction; construction; score; forecast*

Resumen

Las estrategias de los licitadores pueden ser diferentes dependiendo de la Fórmula Económica de Puntuación empleada en una licitación y de la ponderación del precio sobre el total de la puntuación de la oferta.

En este artículo se presenta un nuevo Modelo de Predicción de Bajas. Las diferentes relaciones matemáticas y las funciones de densidad que describen los principales Parámetros de Puntuación y de Predicción, así como la representación de históricos de licitaciones por medio de Gráficos de Curvas de Isopuntuación, permiten la generación de dos nuevos tipos de gráficos que pueden ser muy útiles para los licitadores a la hora de definir sus estrategias para la contratación.

Palabras clave: *oferta; licitación; subasta; construcción; puntuación; predicción*

1. Introducción

The economical criterion is usually one of the most important evaluation criteria and in order to rate different proposals Economic Scoring Formula (ESF) are used. The variables of those formulae are called in this paper Scoring Parameters (SP). In order to foresee SPs, some other parameters, which are called Forecasting Parameters (FP), must be accounted. This paper formulates a new Bid Tender Forecasting Model (BTFM) which is based on the relationship between scoring and forecasting parameters (SPs and FPs respectively). Furthermore, the results are plotted onto two new types of graphs: the Scoring Probability

Graph (SPG) and the Position Probability Graph (PPG) together with the aforementioned iso-Score Curves Graphs (iSCG).

The BTFM described in this paper has been derived from capped tenders. Capped tenders' contract value is upper-limited by the owner and it is clearly stated in the tender specifications. Hence, all bidders must underbid that estimation. Nevertheless, the model is not restricted to capped tenders only. When some expressions are reformulated and some new hypothesis added, the model proposed later will be able to be extended to any other type of tenders.

2. Background

Despite the extensive literature on the theory of auctions and competitive bidding for contract tendering, most of the models are based on theoretical assumptions that are difficult to apply to real cases (Skitmore, 2008). Bidding theory and strategy models (see Stark and Rothkopf, 1979, for an early bibliography) frequently make use of the so-called 'the statistical hypotheses' as auction bids are assumed to contain statistical properties such as fixed parameters and randomness (Skitmore, 2002).

Different models have been developed to calculate the probability, *Pr(m)*, of individual contestants winning a bidding auction (Skitmore et al., 2007); some of such models are Friedman's (Friedman, 1956), Gates' (Gates, 1967), Carr's (Carr, 1982) and Skitmore's (Skitmore, 1991) models, among others. These models are based on the same statistical model differing only in their method of parameter estimation.

In the context of construction contract bidding, it is difficult to collect the necessary data of each bidder for predictions to be effective (Skitmore, 2002). Besides, Skitmore showed that the homogeneity assumption (Skitmore, 1991) was not a valid approach for predicting the probability of lowest bidders (Skitmore, 2002) and Runeson and Skitmore (1999) criticized the use heterogeneous models based on fixed parameters.

The BTFM proposed in this paper solves the main problems encountered in previous models (Skitmore & Runeson, 2006) as it enables (1) to study bidding behaviors with a significant small database compared to previous works; (2) to forecast the probability of obtaining a particular score and/or position among competitors, and (3) to analyze time variations between tenders.

Finally, though other formal and analytical risk models have recently been developed to prescribe how risk is to be incorporated into construction bids (Hartono & Yap, 2011; Mohamed et al., 2011; Oo et al., 2008a, 2008b), in practice, price risks are usually excluded from the final bid to improve competitiveness (Laryea & Hughes, 2011). The BTFM that will be developed does not consider risk issues yet, then, risk models can be a useful complement to the proposed forecasting model.

3. Tendering specifications review

Ballesteros-Pérez, (2010), reviewed 120 tender specifications documents of Spanish Public Administrations and private companies in order to study the scoring parameters (SPs) and Economic Scoring Formulas (ESFs) in Spain.

The review is quite representative of the Spanish tendering system, as it comprises: tenders and auctions from public administrations (City Councils, local councils, semi-public entities, universities, ministries…), a great variety of civil engineering works and services, representation of different geographical regions (including the islands) and a wide range of Tender Amounts. Although the sample only contains Spanish tender documents, the variables analyzed here are directly applicable to any country where the Administration sets up an initial Tender Amount (A) against which, candidates will underbid (capped tendering or upper-limited-price tendering).

Among the wide range of tender documents collected, several Public Administrations generated a large enough number of tendering processes to permit an in-depth statistical analysis. Although the results obtained from those Public Administrations were very similar, a sub-dataset from a particular Public Administration was selected in order to illustrate through a numeric example the mathematical relationships between the SPs and one FP (Estimated Cost or D_0 , in this case).

4. Drawing up the Scoring Probability Graph (SPG)

To draw up the SPG, the following 5-step procedure is proposed (Ballesteros-Pérez, 2010):

1. Get and screen previous tenders as similar as possible with the tender to be forecasted.
2. Calculate SPs and FPs regression coefficients.
3. Specify the Estimated Cost (D_0) into SP's regression equations
4. Draw up the iso-Score Curves Graph (iSCG)
5. Draw up the Scoring Probability Graphs (SPG)

These five steps will be explained in detail while developing a numeric example based upon a real ACA's tendering dataset.

4.1 Analysis of previous tenders

An historic dataset is necessary for any forecast. Otherwise it is rather difficult to work out a proper prediction. Every tender must include a register of, at least, the following data: Tender Code / ID; Tender deadline; Nature of work, Economic Tender amount, Number of Bidders (N), Mean Drop (D_m), Maximum Drop (D_{max}), Minimum Drop (D_{min}), Bidders' Bids' standard deviation (σ) and Estimated Cost (D_0).

Furthermore, concerning BTFM it is necessary to start from a collection of previous and homogeneous tenders with the one is going to be forecasted. By the term "homogeneous" we understand that previous and future tenders must share the same or very similar: scope of works, ESF and geographic region.

Table 1: ACA's Historic Tender sub-dataset

Tender Code	ID	Tender Deadline	Nature of Work	Tender Amount (€)	Dmax	Dm	Dmin	σ	Do	a (Dmax)	b (Dmin)	c(σ)	d (Do)	N
CT08000389	8	2008-03-31	WWTP	4,745,844.66 €	0.2732	0.1813	0.0514	0.0491	0.1170	-0.6193	0.8751	0.1277	1.0786	22
CT07002822	18	2008-01-23	WWTP	2,279,367.16 €	0.2367	0.1854	0.1028	0.0381	0.0700	-0.3398	0.5470	0.0991	1.1416	14
CT07002921	19	2008-01-23	WWTP	4,346,995.62 €	0.2833	0.2262	0.1818	0.0383		-0.3268	0.2533	0.1001		14
CT07002108	24	2007-12-11	WWTP	5,208,624.36 €	0.2390	0.1765	0.0955	0.0386	0.1210	-0.4301	0.5573	0.1003	1.0674	22
CT07001934	27	2007-09-17	WWTP	6,557,087.95 €	0.2200	0.1277	0.0325	0.0530	0.1120	-0.8282	0.8547	0.1409	1.0181	16
CT07001972	28	2007-09-17	WWTP	8,764,690.65 €	0.2800	0.2097	0.1550	0.0428		-0.4246	0.3299	0.1112		10
CT07002052	29	2007-09-17	WWTP	6,217,700.13 €	0.2985	0.2361	0.1489	0.0561		-0.3459	0.4837	0.1468		9
CT07001903	33	2007-08-30	WWTP	2,773,494.15 €	0.1500	0.0855	0.0105	0.0523	0.0350	-0.8248	0.9592	0.1474	1.0552	9
CT07001602	36	2007-08-20	WWTP	3,489,863.47 €	0.2450	0.1199	0.0374	0.0666		-1.1863	0.7816	0.1785		9
										a_m	b_m	c_m	d_m	N_m
									Average :	-0.59	0.63	0.13	1.07	13.89
										$a_σ$	$b_σ$	$c_σ$	$d_σ$	$N_σ$
									Desvest :	0.30	0.25	0.03	0.05	5.28

The bidder which plans to make forecasting must have taken part in a minimum of three tenders analyzed in order to be able to calculate the connection between FPs and SPs, so some previous tenders' Estimated Costs can be known (D_0 values).

In this example, a bidder intends to bid the construction of a WWTP with a tender amount around 4.5 million euros. The tender has been published by the public Spanish Administration "ACA" and its deadline is due on June 2008. With these data we would be able to use a real historic and homogeneous 9- tender sub-dataset as presented on Table 1.

It can be observed that their deadlines, nature of work and tender amounts are very similar to the one to be forecasted.

The ESF and the ALBC used by the Administration for these tenders was always the same and coincident with the future tender's one (see Appendix A to look up each variable's meaning) :

ESF: $S_i = 1 - \frac{B_i - B^*_{min}}{B_{max} - B^*_{min}}$ ALBC: $B_{abn} = B_m - 2\,S$

4.2 Calculation of SP and FP regression coefficients

Once the previous tenders' dataset is available and it has been filtered to select past tenders as similar as possible with the tender which is going to be forecasted the next step is to know what particular regression coefficients' values have the mathematical relationships which interconnect SPs and FPs (See Appendix B, Regression equations between SPs and FPs).

These coefficients are named: *a*, *b*, *c* and *d*, and their partial and final results are shown on the last five Table 1's columns (the mathematical expressions applied are exposed in Appendix B, Expressions for calculating Regression equations' coefficients' values). In Ballesteros-Pérez, 2010 it was demonstrated that these coefficients' variation follows a Normal distribution, so their dispersion can be studied by means of their respective averages and standard deviations.

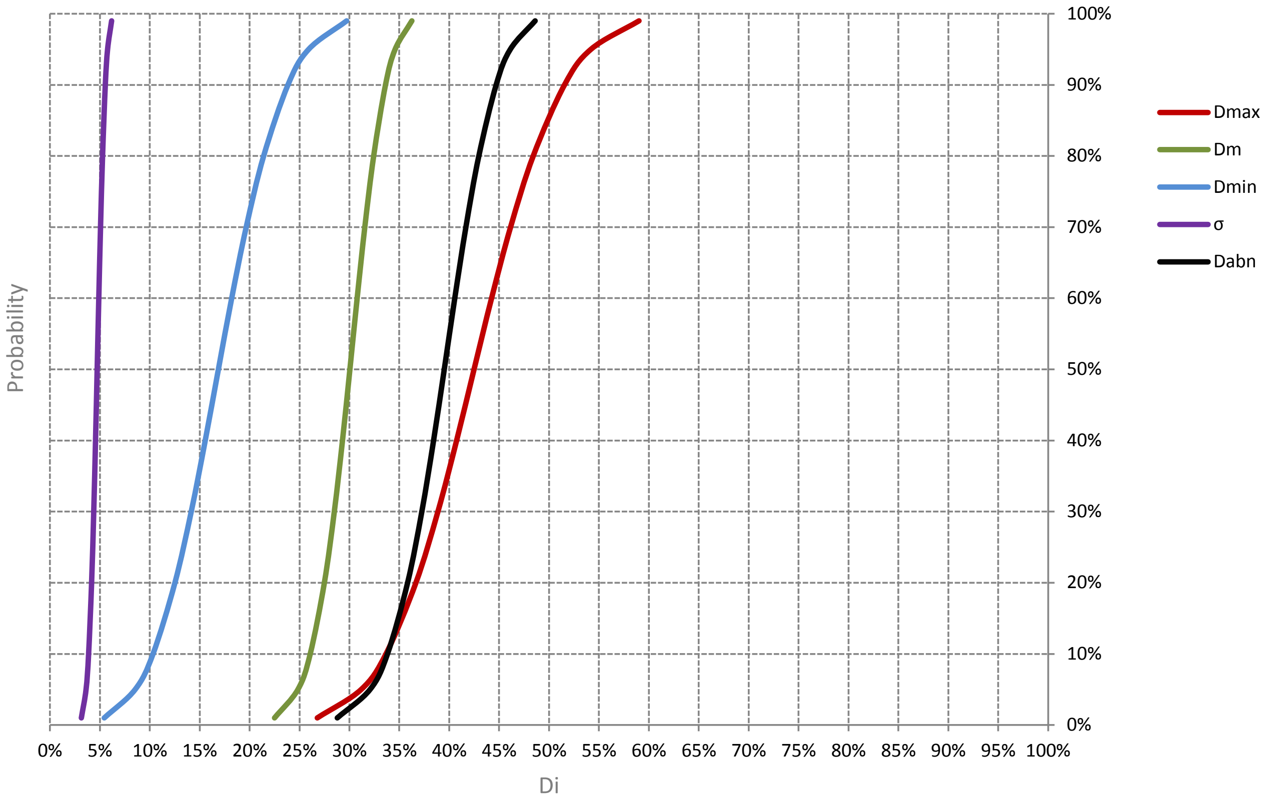
4.3 Future tender's Estimated Cost (D_0) specification

Regarding this variable, the only thing we consider to be significant is that it must always be calculated in the same way, which means: being calculated by the same person or the same group of people with the same criteria; and aggregating the same type of costs every time (taxes, indirect costs, structure cost from the company…). If a mark-up were included in D_0, the following occasions D_0 were calculated it should include the same mark-up percentage.

Finally, the type of works and the ESF must be the same in every case. When these items suffer an important change, the historical data of D_0 will be deleted and it will be necessary to start from scratch correlating D_0 with the SPs for future tenders, i.e., determining *a*, *b*, *c* and *d* values again.

Once the future tender's D_0 is calculated (in our example we will admit that the estimated cost was equal to D_0=0.25) by the bidder which is forecasting, its value must be introduced in the BTFM's regression expressions (see Appendix B, BTFM's Regression expressions as a function of D_0 and Appendix B, Expressions for calculating asterisk regression coefficients' values). Table 2 and Figure 1 shows the main results, where "n_σ" represents the multiples of Standard Deviation that are related to a particular accumulated Standard Normal Distribution probability (axis Y) and "h" represents a coefficient that takes into account the number of dimensions involved in the multivariate Normal Distribution (see Appendix A).

Figure 1: Dm's, Dmax's, Dmin's, σ's and Dabn's Probability Curves for D0=0.25



On Table 2, Scoring Parameter D_{abn} has also been calculated. Its values depended exclusively on D_m and σ for the ALBC exposed as an example ($D_{abn} = D_m + 2\sigma$; once it has been transformed into Drops).

Table 2: Dm's, Dmax's, Dmin's, σ's and Dabn's Probability Curves' Calculation for D0=0.25

Probability (axis Y)	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.99

BTFM's Dm's regression curves													
d_m	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07
h (1 variable total)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n_σ	-2.32	-1.65	-1.28	-0.84	-0.52	-0.25	0.00	0.25	0.52	0.84	1.28	1.65	2.32
d_σ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$d^*=d_m+h\cdot n_\sigma\cdot d_\sigma$	0.97	1.00	1.01	1.03	1.05	1.06	1.07	1.08	1.10	1.11	1.13	1.15	1.18
Dm (axis X)	**0.22**	**0.25**	**0.26**	**0.27**	**0.28**	**0.29**	**0.30**	**0.31**	**0.32**	**0.32**	**0.34**	**0.35**	**0.36**

BTFM's Dmax's regression curves													
a_m	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59
h (2 variables total)	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
n_σ	2.32	1.65	1.28	0.84	0.52	0.25	0.00	-0.25	-0.52	-0.84	-1.28	-1.65	-2.32
a_σ	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
$a^*=a_m+h\cdot n_\sigma\cdot a_\sigma$	-0.10	-0.24	-0.32	-0.41	-0.48	-0.54	-0.59	-0.64	-0.70	-0.77	-0.86	-0.94	-1.08
d_m	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07
h (2 variables total)	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
n_σ	-2.32	-1.65	-1.28	-0.84	-0.52	-0.25	0.00	0.25	0.52	0.84	1.28	1.65	2.32
d_σ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$d^*=d_m+h\cdot n_\sigma\cdot d_\sigma$	1.00	1.02	1.03	1.05	1.06	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.15
Dmax (axis X)	**0.27**	**0.31**	**0.34**	**0.37**	**0.39**	**0.41**	**0.42**	**0.44**	**0.46**	**0.48**	**0.52**	**0.54**	**0.59**

BTFM's Dmin's regression curves													
b_m	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63
h (2 variables total)	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
n_σ	2.32	1.65	1.28	0.84	0.52	0.25	0.00	-0.25	-0.52	-0.84	-1.28	-1.65	-2.32
b_σ	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$b^*=b_m+h\cdot n_\sigma\cdot b_\sigma$	1.04	0.92	0.85	0.78	0.72	0.67	0.63	0.58	0.53	0.48	0.40	0.33	0.21
d_m	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07
h (2 variables total)	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
n_σ	-2.32	-1.65	-1.28	-0.84	-0.52	-0.25	0.00	0.25	0.52	0.84	1.28	1.65	2.32
d_σ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$d^*=d_m+h\cdot n_\sigma\cdot d_\sigma$	1.00	1.02	1.03	1.05	1.06	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.15
Dmin (axis X)	**0.05**	**0.09**	**0.10**	**0.13**	**0.14**	**0.16**	**0.17**	**0.18**	**0.20**	**0.21**	**0.24**	**0.26**	**0.30**

BTFM's σ's regression curves													
c_m	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
h (2 variables total)	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
n_σ	-2.32	-1.65	-1.28	-0.84	-0.52	-0.25	0.00	0.25	0.52	0.84	1.28	1.65	2.32
c_σ	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
$c^*=c_m+h\cdot n_\sigma\cdot c_\sigma$	0.08	0.10	0.10	0.11	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.16	0.17
d_m	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07
h (2 variables total)	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
n_σ	-2.32	-1.65	-1.28	-0.84	-0.52	-0.25	0.00	0.25	0.52	0.84	1.28	1.65	2.32
d_σ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$d^*=d_m+h\cdot n_\sigma\cdot d_\sigma$	1.00	1.02	1.03	1.05	1.06	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.15
σ (axis X)	**0.03**	**0.04**	**0.04**	**0.04**	**0.04**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.06**	**0.06**	**0.06**

BTFM's Dabn's curves													
D_m	0.22	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.32	0.34	0.35	0.36
σ	0.03	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06
Dabn (axis X)	**0.29**	**0.32**	**0.34**	**0.36**	**0.37**	**0.38**	**0.40**	**0.41**	**0.42**	**0.43**	**0.45**	**0.46**	**0.49**

4.4 Draw up the iso-Score Curves Graph (iSCG)

In Ballesteros-Pérez, 2010 it is explained how an iSCG is built. Applied to our example, the procedure to generate iso-Score Curves from the ESF is as follows:

1. Express mathematically the Economic Scoring formula. In our example this would lead to:

$$S_i = 1 - \frac{B_i - B^*_{min}}{B_{max} - B^*_{min}} \qquad\qquad B_{abn} = B_m - 2\,S$$

2. Convert the ESF (when expressed in monetary units) into Drops (with parameters expressed in per-unit values):

$$S_i = 1 - \frac{D^*_{max} - D_i}{D^*_{max} - D_{min}} \qquad\qquad D_{abn} = D_m + 2\sigma$$

3. Work out the ESF's value of variable Di (Bidders' Drop):

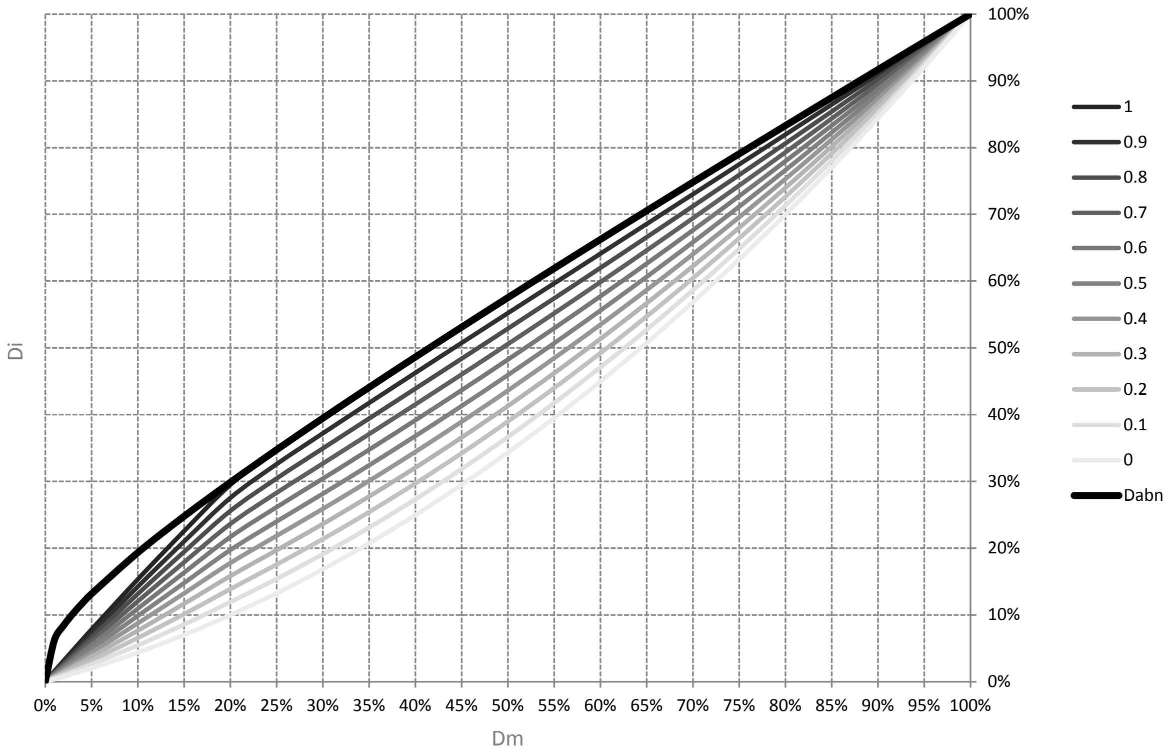
$$D_i = D^*_{max} - (1 - S_i)(D^*_{max} - D_{min})$$

Represent the different iso-Score Curves graphically for the required and/or selected score values (S_i). In our example, Table 3 shows the Scores (S_i) from 0 to 1, place equidistantly at 0,10 intervals (using the first three equations shown in *Appendix B, Regression equations between SPs and FPs*, and the expression obtained in step 3).

Table 3: Calculations of the iso-Score Curves Graph for the example's ESF and ALBC

SPs					D*max	Si (iso-Scoring Curves)										
Dm	Dmax	Dmin	σ	Dabn	min(Dmax;Dabn)	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.01	0.02	0.00	0.03	0.06	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
0.02	0.03	0.01	0.03	0.08	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01
0.03	0.05	0.01	0.04	0.10	0.05	0.05	0.04	0.04	0.04	0.03	0.03	0.03	0.02	0.02	0.02	0.01
0.04	0.06	0.02	0.04	0.12	0.06	0.06	0.06	0.05	0.05	0.04	0.04	0.03	0.03	0.03	0.02	0.02
0.05	0.08	0.02	0.04	0.13	0.08	0.08	0.07	0.07	0.06	0.05	0.05	0.04	0.04	0.03	0.03	0.02
0.10	0.15	0.04	0.05	0.19	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.08	0.07	0.05	0.04
0.15	0.23	0.07	0.05	0.25	0.23	0.23	0.21	0.19	0.18	0.16	0.15	0.13	0.12	0.10	0.09	0.07
0.20	0.29	0.10	0.05	0.30	0.29	0.29	0.28	0.26	0.24	0.22	0.20	0.18	0.16	0.14	0.12	0.10
0.25	0.36	0.13	0.05	0.35	0.35	0.35	0.33	0.30	0.28	0.26	0.24	0.22	0.20	0.18	0.15	0.13
0.30	0.42	0.17	0.05	0.39	0.39	0.39	0.37	0.35	0.33	0.30	0.28	0.26	0.24	0.21	0.19	0.17
0.35	0.48	0.21	0.05	0.44	0.44	0.44	0.42	0.39	0.37	0.35	0.32	0.30	0.28	0.25	0.23	0.21
0.40	0.54	0.25	0.04	0.49	0.49	0.49	0.46	0.44	0.42	0.39	0.37	0.34	0.32	0.30	0.27	0.25
0.45	0.60	0.29	0.04	0.53	0.53	0.53	0.51	0.48	0.46	0.44	0.41	0.39	0.37	0.34	0.32	0.29
0.50	0.65	0.34	0.04	0.58	0.58	0.58	0.55	0.53	0.51	0.48	0.46	0.44	0.41	0.39	0.37	0.34
0.55	0.70	0.39	0.03	0.62	0.62	0.62	0.60	0.57	0.55	0.53	0.51	0.48	0.46	0.44	0.42	0.39
0.60	0.74	0.45	0.03	0.66	0.66	0.66	0.64	0.62	0.60	0.58	0.56	0.53	0.51	0.49	0.47	0.45
0.65	0.78	0.51	0.03	0.71	0.71	0.71	0.69	0.67	0.65	0.63	0.61	0.59	0.57	0.55	0.53	0.51
0.70	0.82	0.57	0.02	0.75	0.75	0.75	0.73	0.71	0.69	0.68	0.66	0.64	0.62	0.60	0.59	0.57
0.75	0.86	0.63	0.02	0.79	0.79	0.79	0.77	0.76	0.74	0.73	0.71	0.70	0.68	0.66	0.65	0.63
0.80	0.89	0.70	0.02	0.83	0.83	0.83	0.82	0.81	0.79	0.78	0.77	0.75	0.74	0.73	0.71	0.70
0.85	0.93	0.77	0.01	0.87	0.87	0.87	0.86	0.85	0.84	0.83	0.82	0.81	0.80	0.79	0.78	0.77
0.90	0.95	0.84	0.01	0.92	0.92	0.92	0.91	0.90	0.89	0.89	0.88	0.87	0.87	0.86	0.85	0.84
0.95	0.98	0.92	0.00	0.96	0.96	0.96	0.95	0.95	0.95	0.94	0.94	0.94	0.93	0.93	0.92	0.92
1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Figure 2: iso-Score Curves Graph for the example's ESF and ALBC



It must be pointed out that the ESF proposed by the Administration in our example gives the maximum score (Si) to the Bidder which proposed a maximum but not abnormally high Drop.

That is the reason why D^{*}_{max} is equal to the minimum value between D_{max} and D_{abn} for any possible probability level.

If Table 3 is represented as a function of variable D_m, the iSCG is finally drawn up and the first of the BTFM's graphs achieved.

4.5 Draw up the Scoring Probability Graph (SPG)

Once the ESF is work out as a function of D_i , D_m's, D_{max}'s, D_{min}'s, σ's and D_{abn}'s Probability Curves' values from Table 2 (figures in bold) or Figure 1 can be introduced in that expression, obtaining the lower part of Table 4 with ease.

Table 4: Calculations of the SPG for the example's ESF and ALBC for D0=0.25

↓SP Prob→	**0.01**	**0.05**	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**0.99**
Dmax	0.27	0.31	0.34	0.37	0.39	0.41	0.42	0.44	0.46	0.48	0.52	0.54	0.59
Dm	0.22	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.32	0.34	0.35	0.36
Dmin	0.05	0.09	0.10	0.13	0.14	0.16	0.17	0.18	0.20	0.21	0.24	0.26	0.30
σ	0.03	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06
Dabn	0.29	0.32	0.34	0.36	0.37	0.38	0.40	0.41	0.42	0.43	0.45	0.46	0.49
D*max	0.27	0.31	0.34	0.36	0.37	0.38	0.40	0.41	0.42	0.43	0.45	0.46	0.49

↓Si Prob→	**0.01**	**0.05**	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**0.99**
1.00	0.27	0.31	0.34	0.36	0.37	0.38	0.40	0.41	0.42	0.43	0.45	0.46	0.49
0.90	0.25	0.29	0.31	0.33	0.35	0.36	0.37	0.38	0.39	0.41	0.43	0.44	0.47
0.80	0.23	0.27	0.29	0.31	0.33	0.34	0.35	0.36	0.37	0.39	0.41	0.42	0.45
0.70	0.20	0.24	0.27	0.29	0.30	0.32	0.33	0.34	0.35	0.36	0.38	0.40	0.43
0.60	0.18	0.22	0.24	0.27	0.28	0.29	0.30	0.32	0.33	0.34	0.36	0.38	0.41
0.50	0.16	0.20	0.22	0.24	0.26	0.27	0.28	0.29	0.31	0.32	0.34	0.36	0.39
0.40	0.14	0.18	0.20	0.22	0.23	0.25	0.26	0.27	0.28	0.30	0.32	0.34	0.37
0.30	0.12	0.15	0.17	0.20	0.21	0.22	0.24	0.25	0.26	0.28	0.30	0.32	0.35
0.20	0.10	0.13	0.15	0.17	0.19	0.20	0.21	0.23	0.24	0.26	0.28	0.30	0.34
0.10	0.08	0.11	0.13	0.15	0.16	0.18	0.19	0.20	0.22	0.24	0.26	0.28	0.32
0.00	0.05	0.09	0.10	0.13	0.14	0.16	0.17	0.18	0.20	0.21	0.24	0.26	0.30

Figure 3: SPG in which curves represent different possible Scores

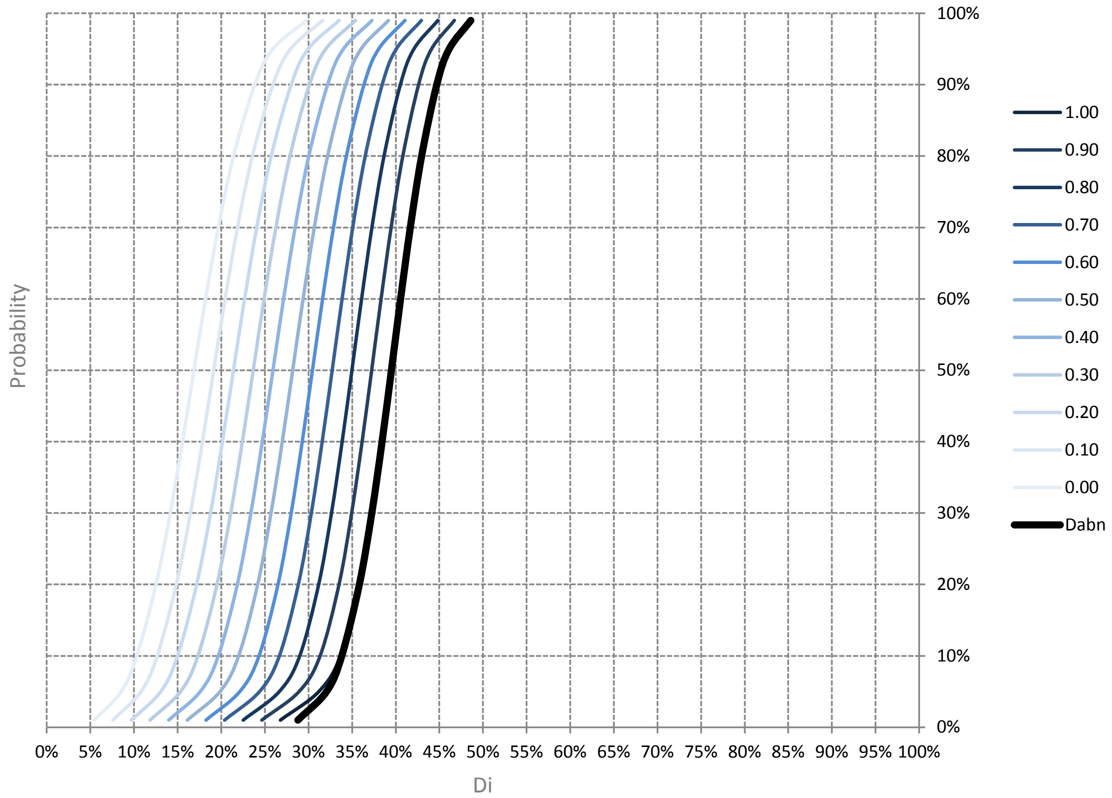


Figure 4: SPG in which curves represent different probable Scoring distribution curves

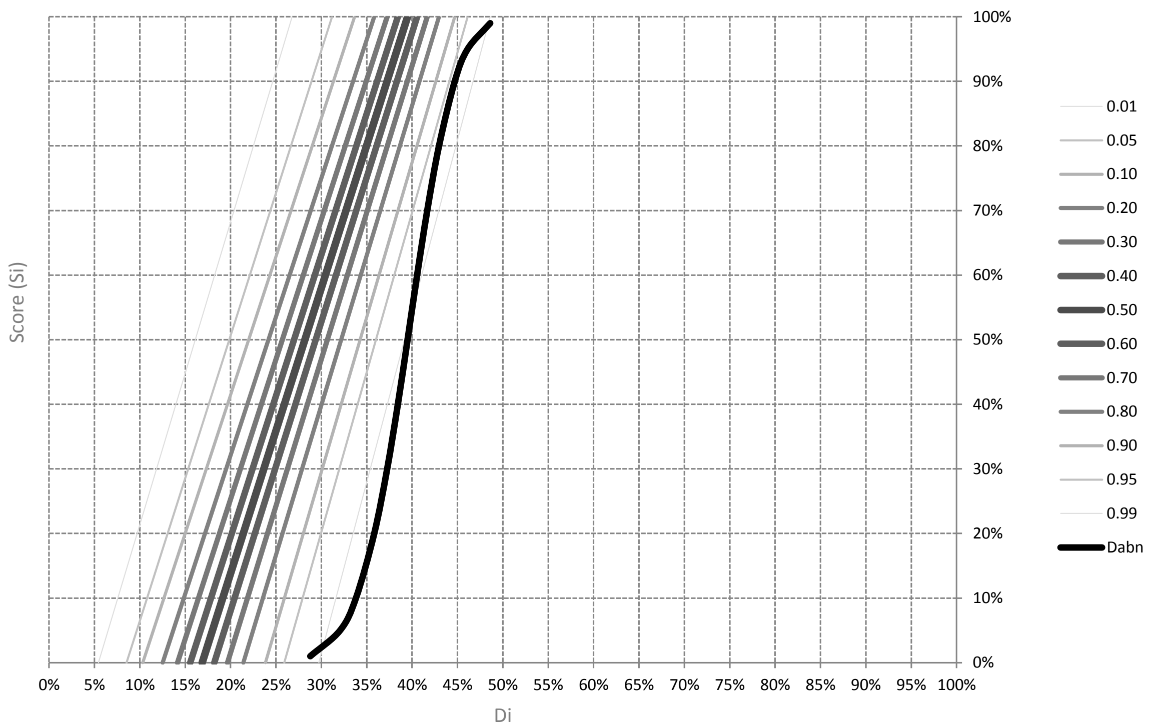


Figure 3's graph allows any bidder to know, for every possible Drop (price discount, on axis X), the probabilities it has of surpassing any possible score and of surpassing the abnormally high drop threshold. This graph constitutes the core of the BTFM, since its simplicity allows an easy reading while including valuable processed tendering data.

Figure 4's graph represents again the same variable on axis X, i.e., D_i., but this time, Y-axis represents scores, which means that every curve shown (but for the D_{abn} curve which coincides with the previous SPG) represents the different probable Scoring distribution curves that the final actual group of bidders' bids is likely to produce (the more thick a curve is the more likely the real Scoring distribution curve will occupy that position once the bidders' bid are opened and known).

5. Drawing up the Position Probability Graph (PPG)

PPG constitutes a further step once the previous graphs have been calculated.

5.1 Number of potential bidders' estimation

To be able to determine the probable positions every competitor will occupy it is necessary to delimit the number of potential bidders that will probably bid in the future tender. The BTFM will consider the variable "Number of Bidders (N)" as a random variable.

The following step is to study the Bidders' Bids distances with each other, so the different competitors' positions can be forecasted. The BTFM has two advantages other previous models did not have: first, the limits of the maximum (D_{max}) and minimum (D_{min}) drops can be statistically determined, and, second, it is known the Drop value that will split up half of the bidders above and below, and that is D_m. An assumption must be made: inside each range, the N/2 bidders will be placed equidistantly at the same probability intervals. Figure 5 shows the relationship between bidders' position (n) and Probability of surpassing the bidders' drops (P_{nth}).

Figure 5: Diagram showing the relationship between n and Pnth.

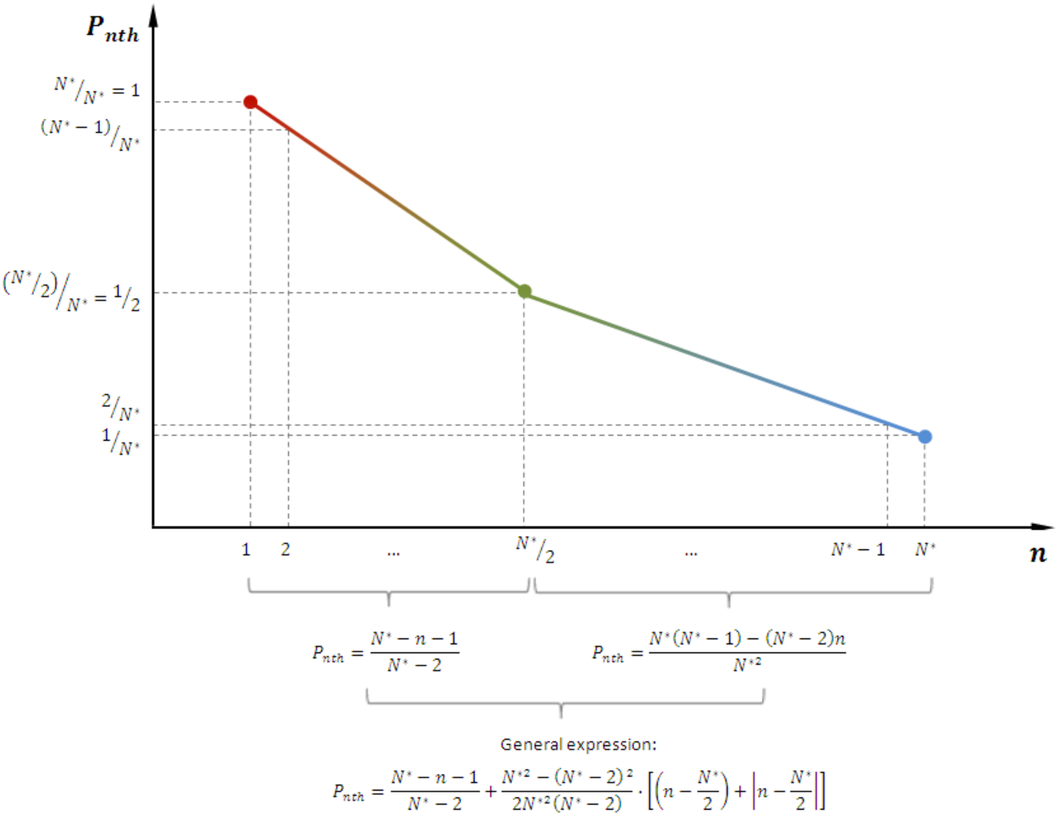


Figure 6: Diagram showing the relationship between Dnth and Pnth.

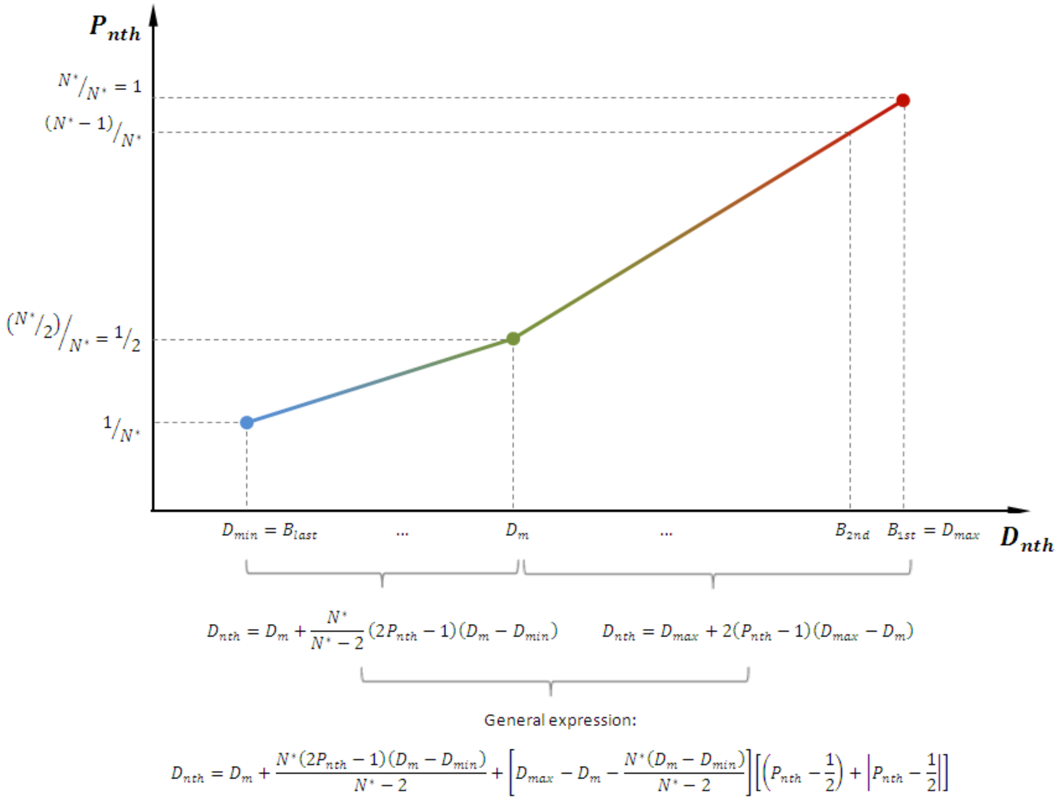


Figure 6 shows the relationship between the bidders' drops (D_{nth}) and the aforementioned probability P_{nth}.

5.2 Calculating and representing Bidders' position curves

Once variables D_{min}, D_m ,D_{max} and N* are known for every possible probability level, and it is assumed that the potential number of bidders, whatever it is, will follow a bi-linear distribution as it was represented on figure 6, remains calculating each bidder n's position curve applying the general expressions shown at the bottom of figures 5 and 6.

Calculations for the first 25 potential bidders' positions are shown on Table 5 and its representation appears on Figure 7. The only precaution that it must be taken is not to use N* values lower than 2, in order to maintain equations' consistency.

Figure 7: Position Probability Graph for the example's ESF and ALBC and for D0=0.25

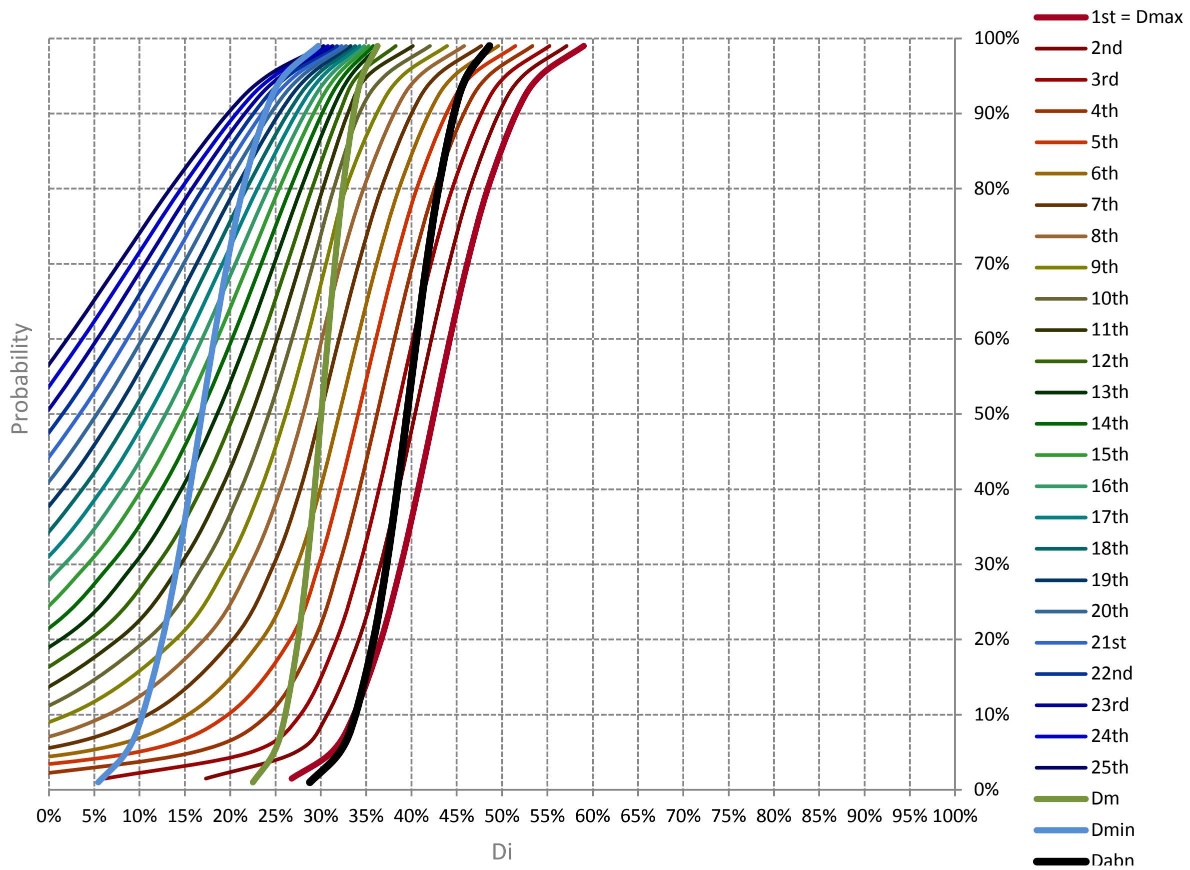


Table 5: Calculations of the PPG for the example's ESF and ALBC for D0=0.25

This last kind of graph enables any bidder to study which positions are the most interesting ones to be occupied as a function of any possible Drop (Bid), since, quite often, first positions involve high risks of being disqualified because of the ALBC (as it happens to the first and second bidders represented on figure 7 because tthey have most part of their curves on the right of D_{abn}).

In the example analyzed, a third, fourth or even fifth position would be a the more convenient since they would turn to first, second and third positions respectively once the riskier bidders were eliminated. Therefore, this graph gives complementary information regarding the data given by the Scoring Probability Graphs.

↓SP Prob→	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.99
Dmax	0.27	0.31	0.34	0.37	0.39	0.41	0.42	0.44	0.46	0.48	0.52	0.54	0.59
Dm	0.22	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.32	0.34	0.35	0.36
Dmin	0.05	0.09	0.10	0.13	0.14	0.16	0.17	0.18	0.20	0.21	0.24	0.26	0.30
Dabn	0.29	0.32	0.34	0.36	0.37	0.38	0.40	0.41	0.42	0.43	0.45	0.46	0.49

Total number of Bidders' Estimate (N*)													
N_m	13.89	13.89	13.89	13.89	13.89	13.89	13.89	13.89	13.89	13.89	13.89	13.89	13.89
h (1 variable total)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n_σ	-2.05	-1.65	-1.28	-0.84	-0.52	-0.25	0.00	0.25	0.52	0.84	1.28	1.65	2.32
N_σ	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28
$N^*=N_m+h\cdot n_\sigma\cdot N_\sigma$	3.07	5.18	7.13	9.46	11.14	12.57	13.89	15.21	16.63	18.32	20.65	22.60	26.13

↓ Bidders	↓n Bidder	P_{nth}												
1st = Dmax	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2nd	2	0.45	0.69	0.81	0.87	0.89	0.91	0.92	0.92	0.93	0.94	0.95	0.95	0.96
3rd	3	0.33	0.45	0.61	0.73	0.78	0.81	0.83	0.85	0.86	0.88	0.89	0.90	0.92
4th	4	0.22	0.33	0.46	0.60	0.67	0.72	0.75	0.77	0.79	0.82	0.84	0.85	0.88
5th	5	0.11	0.21	0.36	0.48	0.56	0.62	0.66	0.70	0.73	0.75	0.79	0.81	0.83
6th	6	-0.01	0.10	0.25	0.39	0.47	0.53	0.58	0.62	0.66	0.69	0.73	0.76	0.79
7th	7	-0.12	-0.02	0.15	0.31	0.39	0.45	0.50	0.55	0.59	0.63	0.68	0.71	0.75
8th	8	-0.23	-0.14	0.05	0.23	0.32	0.39	0.43	0.48	0.52	0.57	0.62	0.66	0.71
9th	9	-0.35	-0.26	-0.05	0.14	0.25	0.32	0.37	0.42	0.46	0.51	0.57	0.61	0.67
10th	10	-0.46	-0.38	-0.15	0.06	0.17	0.25	0.31	0.36	0.41	0.46	0.52	0.56	0.63
11th	11	-0.57	-0.50	-0.25	-0.02	0.10	0.18	0.25	0.31	0.36	0.41	0.47	0.51	0.59
12th	12	-0.69	-0.62	-0.35	-0.11	0.03	0.12	0.19	0.25	0.31	0.36	0.43	0.47	0.54
13th	13	-0.80	-0.73	-0.45	-0.19	-0.05	0.05	0.13	0.19	0.25	0.31	0.38	0.43	0.50
14th	14	-0.91	-0.85	-0.55	-0.27	-0.12	-0.02	0.07	0.13	0.20	0.26	0.34	0.39	0.47
15th	15	-1.03	-0.97	-0.65	-0.36	-0.19	-0.08	0.00	0.08	0.15	0.22	0.30	0.35	0.43
16th	16	-1.14	-1.09	-0.75	-0.44	-0.27	-0.15	-0.06	0.02	0.09	0.17	0.25	0.31	0.40
17th	17	-1.25	-1.21	-0.86	-0.52	-0.34	-0.22	-0.12	-0.04	0.04	0.12	0.21	0.27	0.36
18th	18	-1.37	-1.33	-0.96	-0.61	-0.42	-0.28	-0.18	-0.09	-0.01	0.07	0.16	0.23	0.33
19th	19	-1.48	-1.44	-1.06	-0.69	-0.49	-0.35	-0.24	-0.15	-0.07	0.02	0.12	0.19	0.29
20th	20	-1.60	-1.56	-1.16	-0.77	-0.56	-0.42	-0.30	-0.21	-0.12	-0.03	0.08	0.15	0.26
21th	21	-1.71	-1.68	-1.26	-0.86	-0.64	-0.48	-0.37	-0.26	-0.17	-0.08	0.03	0.11	0.22
22th	22	-1.82	-1.80	-1.36	-0.94	-0.71	-0.55	-0.43	-0.32	-0.22	-0.12	-0.01	0.07	0.18
23th	23	-1.94	-1.92	-1.46	-1.02	-0.78	-0.62	-0.49	-0.38	-0.28	-0.17	-0.05	0.03	0.15
24th	24	-2.05	-2.04	-1.56	-1.11	-0.86	-0.69	-0.55	-0.44	-0.33	-0.22	-0.10	-0.01	0.11
25th	25	-2.16	-2.16	-1.66	-1.19	-0.93	-0.75	-0.61	-0.49	-0.38	-0.27	-0.14	-0.05	0.08

		D_{nth}												
↓ Bidders	↓n Bidder Prob→	0.02	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.99
1st = Dmax	1	0.27	0.31	0.34	0.37	0.39	0.41	0.42	0.44	0.46	0.48	0.52	0.54	0.59
2nd	2	0.17	0.27	0.31	0.34	0.37	0.39	0.40	0.42	0.44	0.46	0.50	0.52	0.57
3rd	3	0.06	0.22	0.28	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.48	0.50	0.55
4th	4	-0.05	0.16	0.24	0.29	0.32	0.34	0.36	0.38	0.40	0.43	0.46	0.48	0.53
5th	5	-0.16	0.10	0.20	0.27	0.30	0.32	0.34	0.36	0.38	0.41	0.44	0.47	0.51
6th	6	-0.27	0.03	0.15	0.23	0.27	0.30	0.32	0.34	0.36	0.39	0.42	0.45	0.50
7th	7	-0.38	-0.03	0.11	0.20	0.25	0.28	0.30	0.32	0.34	0.37	0.40	0.43	0.48
8th	8	-0.49	-0.09	0.06	0.17	0.22	0.26	0.28	0.30	0.32	0.35	0.38	0.41	0.46
9th	9	-0.61	-0.16	0.02	0.14	0.20	0.23	0.26	0.28	0.31	0.33	0.36	0.39	0.44
10th	10	-0.72	-0.22	-0.02	0.11	0.17	0.21	0.24	0.27	0.29	0.31	0.34	0.37	0.42
11th	11	-0.83	-0.28	-0.07	0.08	0.15	0.19	0.22	0.25	0.28	0.30	0.33	0.35	0.40
12th	12	-0.94	-0.34	-0.11	0.04	0.12	0.17	0.20	0.24	0.26	0.29	0.32	0.34	0.38
13th	13	-1.05	-0.41	-0.16	0.01	0.09	0.15	0.19	0.22	0.25	0.28	0.31	0.33	0.36
14th	14	-1.16	-0.47	-0.20	-0.02	0.07	0.12	0.17	0.20	0.23	0.27	0.30	0.33	0.36
15th	15	-1.27	-0.53	-0.24	-0.05	0.04	0.10	0.15	0.19	0.22	0.25	0.29	0.32	0.35
16th	16	-1.38	-0.60	-0.29	-0.08	0.02	0.08	0.13	0.17	0.21	0.24	0.28	0.31	0.35
17th	17	-1.49	-0.66	-0.33	-0.11	-0.01	0.06	0.11	0.15	0.19	0.23	0.27	0.30	0.34
18th	18	-1.61	-0.72	-0.38	-0.15	-0.03	0.04	0.09	0.14	0.18	0.22	0.26	0.29	0.34
19th	19	-1.72	-0.78	-0.42	-0.18	-0.06	0.01	0.07	0.12	0.16	0.21	0.25	0.29	0.33
20th	20	-1.83	-0.85	-0.47	-0.21	-0.09	-0.01	0.05	0.10	0.15	0.19	0.24	0.28	0.33
21st	21	-1.94	-0.91	-0.51	-0.24	-0.11	-0.03	0.03	0.09	0.13	0.18	0.24	0.27	0.32
22nd	22	-2.05	-0.97	-0.55	-0.27	-0.14	-0.05	0.01	0.07	0.12	0.17	0.23	0.26	0.32
23rd	23	-2.16	-1.04	-0.60	-0.30	-0.16	-0.07	0.00	0.05	0.11	0.16	0.22	0.26	0.31
24th	24	-2.27	-1.10	-0.64	-0.34	-0.19	-0.09	-0.02	0.04	0.09	0.15	0.21	0.25	0.31
25th	25	-2.38	-1.16	-0.69	-0.37	-0.21	-0.12	-0.04	0.02	0.08	0.13	0.20	0.24	0.30

6. Results

A complete Bid Tender Forecasting Model (BTFM) has been developed. So far, none of the previous models have managed to calculate and to present in a brief graphical way as much information as the exposed model does.

The whole BTFM constitutes the sum of four graphs:

1. The iso-Score Curves Graph (iSCG) is able to represent how a particular ESF distributes the economic points in a certain tender and to represent the relationships between the main Scoring Parameters involved. There is no need of previous tender datasets, because it is a merely descriptive tool. Once the user is trained on its use,

the iSCG highlights potential bidding strategies the bidders will probably deploy in order to be more competitive.

2. The Scoring Probability Graphs (SPG), by means of its two representations constitutes the core of the BTFM. These complementary and easy-to-use graphs allow any bidder to measure the probabilities of obtaining any economic score as a function of any possible Bid. Besides they provide indispensable information regarding the likely limits where the Abnormally Low Bid Threshold will be.

The Position Probability Graph (PPG) makes possible the study of the problem from the perspective of the likely positions, that is, it takes into consideration the previous encounters with the competition concerning the number of bidders and its distribution. Taking into account that a minimum economic score is not always enough and that, most of the time, it is necessary to occupy one of the first but “accepted” positions to be competitive enough, this graph enables any bidder to identify which positions must be the more desirable.

7. Conclusions and Discussion

Whereas previous models were based on probabilistic description of large groups of single bidders studying particularly each potential bidder (it was even necessary to know the bidders’ names and have an enormous amount of previous information with regard to their bids), the proposed BTFM describes group patterns as a whole (with a significant smaller dataset) and only when the bidders’ positions must be studied, the model discretizes particular bidders’ behaviors in order to study their more likely positions.

There is a drawback: no analysis can be developed regarding what a specific bidder (a particular company for instance) will probably propose on its economic bid, but, even so, the BTFM proposed solves the major problems previous models have as it allows us to:

1. study bidding behaviors with a significant small database compared to previous works. The iSCG does not need any previous tender experience, and the rest of the graphs can be fully operative with at least three previous tender whenever they are homogeneous with the one which is going to be forecasted (same kind of work, ESF, location and relatively similar budget).
2. forecast the probability of obtaining a particular score and/or position among competitors. The previous models were only able to study the first position alone and they never studied the likely scoring nor the abnormally low bids’ likely thresholds.

analyze time variations between tenders. Although this has not been explained in this paper, it is easy to reach to the conclusion that time variations is easily studied by means of regression coefficients’ (a, b, c and d) variation. If these coefficients’ values are represented on a Y-axis while X-axis represents time, their trends and evolution can be easily identified and quantified.

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Appendix A

Main abbreviations used in the text:

A Amount of money of a Tender (price's upper limitation)

a Regression coefficient to adjust relationship between D_{max} and D_m

a* Variable calculated according to the expression $a_m+h \cdot n_\sigma \cdot a_\sigma$

a_m Average of regression coefficient a values

a_σ Standard deviation of regression coefficient a values

ACA Agencia Catalana del Agua (owner of the tendering sub-dataset analyzed)

ALB Abnormally Low Bid

ALBC Abnormally Low Bid Criteria

B Bid (expressed in monetary value)

b Regression coefficient to adjust relationship between D_{min} and D_m

b* Variable calculated according to the expression $b_m+h\cdot n_\sigma\cdot b_\sigma$

b_m Average of regression coefficient b values

b_σ Standard deviation of regression coefficient b values

B_{abn} Abnormally Low Bid Treshold (expressed in monetary value)

B_i Bidder's i Bid (expressed in monetary value)

B_m Mean Bid (expressed in monetary value)

B_{max} Highest Bid (expressed in monetary value)

B^*_{min} Lowest but not Abnormally Low Bid (expressed in monetary value). It is equal to the maximum value between B_{min} and B_{abn}.

B_{min} Lowest Bid (expressed in monetary value)

B_0 Amount equivalent to the Estimated Cost (expressed in monetary value)

BTFM Bid Tender Forecasting Model

c Regression coefficient to adjust relationship between σ and D_m

c* Variable calculated according to the expression $c_m+h\cdot n_\sigma\cdot c_\sigma$

c_m Average of regression coefficient c values

c_σ Standard deviation of regression coefficient c values

d Regression coefficient to adjust relationship between D_0 and D_m

d* Variable calculated according to the expression $d_m+h\cdot n_\sigma\cdot d_\sigma$

d_m Average of regression coefficient d values

d_σ Standard deviation of regression coefficient d values

D Drop (expressed in per-unit value)

D_{abn} Abnormal Drop (expressed in per-unit value)

D_i Bidder's *i* Drop (expressed in per-unit value)

D_m Mean Drop (expressed in per-unit value)

D_{max} Maximum Drop (expressed in per-unit value)

D^*_{max} Highest but not Abnormally High Drop (expressed in per-unit value). It is equal to the minimum value between D_{max} and D_{abn}.

D_{min} Minimum Drop (expressed in per-unit value)

D_{nth} Bidder's nth Drop (expressed in per-unit value)

D_0 Drop equivalent to the Estimated Cost (expressed in per-unit value)

ESF Economic Scoring Formula

h Multivariate Normal Distribution Coefficient. If there is one variable h=1, and if there are two variables ($h=\sqrt{2}/2\approx 0,71$).

iSC iso-Score Curve

iSCG iso-Score Curve Graph

N Total number of bidders of a particular tender

N^* Variable calculated according to the expression $N_m+h\cdot n_\sigma\cdot N_\sigma$

N_m Average of analyzed N's values

N_σ Standard deviation of analyzed N's values

n_σ Number of Standard Deviation's multiples that are related to a particular accumulated Standard Normal Distribution probability

P Probability (usually expressed in per-unit value)

P_{nth} Probability of surpassing Bidder nth (usually expressed in per-unit value)

S Bids' Standard Deviation (expressed in monetary value)

S_i Score of Bidder *i* (expressed either in points or in per-unit value)

SP Scoring Parameter

SPG Scoring Probability Graph

PPG Position Probability Graph

T Abnormally High Drop Threshold

WWTPWaste Water Treatment Plant

σ Drops' Standard Deviation (expressed in per-unit value)

Appendix B

Main equations linking the different variables used in the text:

Bidder's i Drop $D_i = 1 - \frac{B_i}{A}$

Relationship between SPs' monetary-based and Drop-based values:

$B_m = (1 - D_m)A$; $B_{min} = (1 - D_{max})A$; $B_{max} = (1 - D_{min})A$; $S = \sigma \cdot A$; $B_{abn} = (1 - D_{abn})A$

Regression equations between SPs and FPs:

$D_{max} = aD_m{}^2 + (1-a)D_m$; $D_{min} = bD_m{}^2 + (1-b)D_m$; $\sigma = c(D_m{}^{1/3} - D_m)$; $D_0 = 1 + d(D_m - 1)$

Expressions for calculating Regression equations' coefficients' values:

$a_k = \frac{D_{max\,k} - D_{m\,k}}{D_{m\,k}{}^2 - D_{m\,k}}$; $b_k = \frac{D_{min\,k} - D_{m\,k}}{D_{m\,k}{}^2 - D_{m\,k}}$; $c_k = \frac{\sigma_k}{D_m{}^{1/3} - D_m}$; $d_k = \frac{D_{0\,k} - 1}{D_{m\,k} - 1}$

BTFM's Regression expressions as a function of D_0:

$$D_m = 1 + \frac{D_o - 1}{d^*}$$

$$D_{max} = a^* D_m{}^2 + (1 - a^*)D_m = a^*\left(1 + \frac{D_o - 1}{d^*}\right)^2 + (1 - a^*)\left(1 + \frac{D_o - 1}{d^*}\right)$$

$$D_{min} = b^* D_m{}^2 + (1 - b^*)D_m = b^*\left(1 + \frac{D_o - 1}{d^*}\right)^2 + (1 - b^*)\left(1 + \frac{D_o - 1}{d^*}\right)$$

$$\sigma = c^*\left(D_m{}^{1/3} - D_m\right) = c^*\left[\left(1 + \frac{D_o - 1}{d^*}\right)^{1/3} - \left(1 + \frac{D_o - 1}{d^*}\right)\right]$$

Expressions for calculating asterisk regression coefficients' values:

$a^* = a_m + h \cdot n_\sigma \cdot a_\sigma$; $b^* = b_m + h \cdot n_\sigma \cdot b_\sigma$; $c^* = c_m + h \cdot n_\sigma \cdot c_\sigma$; $d^* = d_m + h \cdot n_\sigma \cdot d_\sigma$

ESF used in the example:

$S_i = 1 - \frac{B_i - B^*_{min}}{B_{max} - B^*_{min}} \rightarrow S_i = 1 - \frac{D^*_{max} - D_i}{D^*_{max} - D_{min}} \rightarrow D_i = D^*_{max} - (1 - S_i)(D^*_{max} - D_{min})$

ALBC used in the example:

$B_{abn} = B_m - 2\,S \rightarrow D_{abn} = D_m + 2\sigma$

Number of Bidders for each Probability level's calculation:

$N^* = N_m + h \cdot n_\sigma \cdot N_\sigma$

BTFM's relationship between probability (P_{nth}), number of bidders (N*) and and position (n):

For ${}^1/_{N^*} \leq P_{nth} \leq {}^1/_2$ and ${}^{N^*}/_2 \leq n \leq N^*$: $P_{nth} = \frac{N^*(N^*-1)-(N^*-2)n}{N^{*2}}$

For ${}^1/_2 \leq P_{nth} \leq 1$ and $1 \leq n \leq {}^{N^*}/_2$: $P_{nth} = \frac{N^*-n-1}{N^*-2}$

General expression (for ${}^1/_{N^*} \leq P_{nth} \leq 1$, $1 \leq n \leq N^*$ and $\forall N^* > 2$)

$$P_{nth} = \frac{N^* - n - 1}{N^* - 2} + \frac{N^{*2} - (N^* - 2)^2}{2N^{*2}(N^* - 2)} \cdot \left[\left(n - \frac{N^*}{2}\right) + \left|n - \frac{N^*}{2}\right|\right]$$

BTFM's relationship between Bidder nth's Drop (D_{nth}), probability (P_{nth}) and number of bidders (N*):

For ${}^1/_{N^*} \leq P_{nth} \leq {}^1/_2$ and $D_{min} \leq D_{nth} \leq D_m$): $D_{nth} = D_m + \frac{N^*}{N^*-2}(2P_{nth} - 1)(D_m - D_{min})$

For ${}^1/_2 \leq P_{nth} \leq 1$ and $D_m \leq D_{nth} \leq D_{max}$): $D_{nth} = D_{max} + 2(P_{nth} - 1)(D_{max} - D_m)$

General expression (for ${}^1/_{N^*} \leq P_{nth} \leq 1$ and $D_{min} \leq D_{nth} \leq D_{max}$)

$$D_{nth} = D_m + \frac{N^*(2P_{nth} - 1)(D_m - D_{min})}{N^* - 2} + \left[D_{max} - D_m - \frac{N^*(D_m - D_{min})}{N^* - 2}\right]\left[\left(P_{nth} - \frac{1}{2}\right) + \left|P_{nth} - \frac{1}{2}\right|\right]$$

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